A Comparison Of Tree Taper Systems

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Forest Biometrician – Retired

Intermountain Forestry Cooperative Annual Meeting

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The Journey


Olympia, WA
Johnson, WA
Moscow, ID
Brush Prairie, WA
Oregon, WI
Milton, WI
Blacksburg, VA
Lufkin, TX
Baton Rouge, LA

Project Objectives

1. IFC Douglas-fir stem analysis data

2. Three taper systems and their input options:

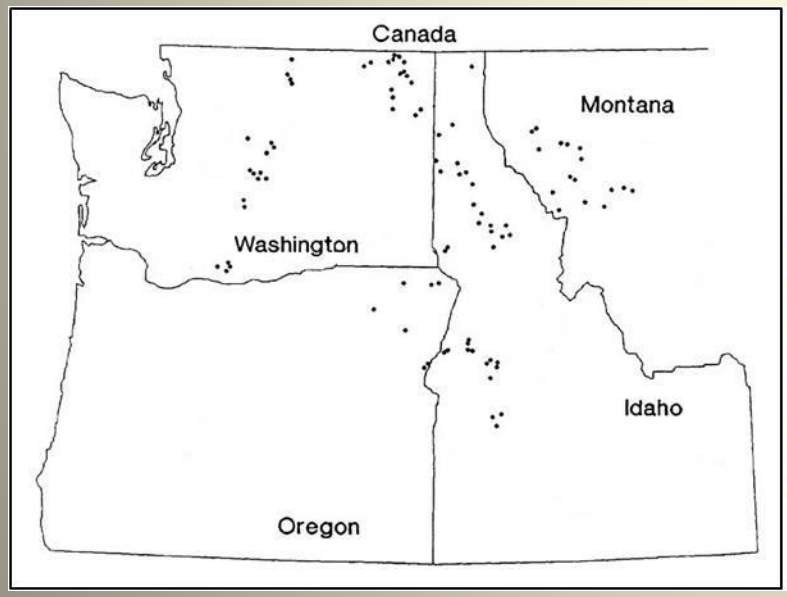
   FPS Taper Class System
   Flewelling's 2 & 3-Point Taper System
   Natural Cubic Spline Interpolation

3. Predict total ft^3 volumes (ob) and heights along the bole using IFC stem analysis data.

4. Generate residual (predicted – observed) statistics and plots for total ft^3 volumes (ob) and heights.

IFC Stem Analysis Data

The IFC destructively sampled 1,085 trees in 94 second-growth Douglas-fir stands. The stands covered a broad range of ages, stand densities, sites, stand conditions, and geographic regions in the Intermountain Northwest.



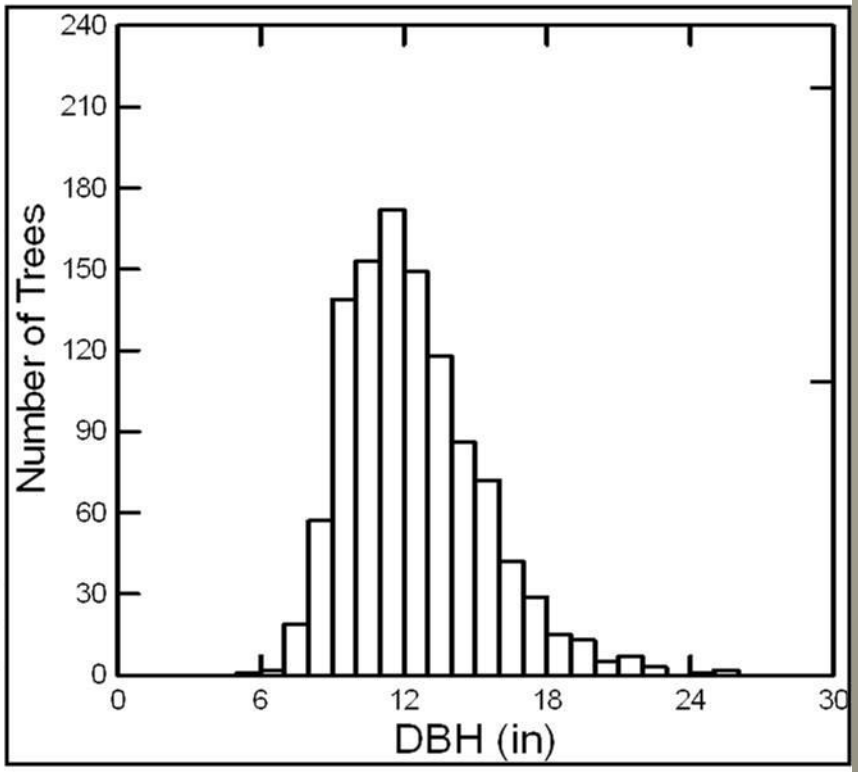
IFC Region	Site Index (ft)								Total
	40	50	60	70	80	90	100	110	
CID	4	53	83	23	0	0	0	0	163
CWA	6	37	56	67	43	18	4	0	231
NEOR	0	10	28	40	15	0	0	0	93
NEWA	2	27	40	56	45	33	4	0	207
NID	0	2	13	23	80	79	15	2	214
WMT	13	21	60	58	23	2	0	0	177
Total	25	150	280	267	206	132	23	2	1,085

IFC Stem Analysis Data

Tree measurements (ob & ib) included stump diameter (1-ft), DBH, total height, diameter and height at the live crown base, and a variable number (3-7) of diameters and heights along the bole.

Tree Statistic	Tree Attribute (n = 1,085)					
	DBH (in)	Total Height (ft)	Crown Ratio (%)	Age (yrs)	Taper Class	Total ft^3 Volume (ob)
Min	5.6	32.1	24	25	0.096	2.8
Max	25.4	124.8	93.4	144	0.447	199.8
Median	12	68	59.2	65	0.266	23.8
Average	12.4	69.8	60.3	64.7	0.268	29.4
Std Dev	2.87	14.46	13.03	18.75	0.059	20.70

DBH Class (in)	Total Height Class (ft)					
	<50	50-70	70-90	90-110	>110	Total
5 - 10	43	163	12	0	0	218
10 - 15	7	379	272	19	1	678
15 - 20	0	16	83	69	3	171
>20	0	0	3	7	8	18
Total	50	558	370	95	12	1,085



IFC Stem Analysis Data

All diameters and heights along the bole were expressed as proportions of DBH and total height, respectively:

$$\text{Relative Diameter} = \frac{\text{dob}}{\text{DBH}} \qquad \text{Relative Height} = \frac{(\text{h} - 4.5)}{(\text{HT} - 4.5)}$$

Taper data for each tree were then standardized using quadratic interpolation with relative diameters at 10% DBH intervals (0-100%) to generate upper stem relative heights.

Measured Tree Data			
DOB (in)	Relative Diameter	Height (ft)	Relative Height
8.30	1.00	4.5	0.000
6.48	0.78	15.0	0.276
4.80	0.58	24.0	0.512
2.62	0.32	33.0	0.748
0.00	0.00	42.6	1.000

Relative Diameter	DOB (in)	Relative Height	Height (ft)
1.0	8.30	0.000	4.5
0.9	7.47	0.131	9.5
0.8	6.64	0.253	14.1
0.7	5.81	0.370	18.6
0.6	4.98	0.487	23.0
0.5	4.15	0.581	26.6
0.4	3.32	0.670	30.0
0.3	2.49	0.760	33.5
0.2	1.66	0.839	36.4
0.1	0.83	0.918	39.5
0.0	0.00	1.000	42.6

Analysis Specifications

1. Tree taper profiles are estimated using observed DBH, total height, and upper stem diameter/height pair(s) to localize/improve estimated stem taper.
2. Taper Class, the relative height along the bole at 80% DBH, is used to localize estimated stem taper.
3. Relative diameters and heights along the bole are expressed as proportions of DBH and total height starting at 4.5 feet.
4. Dob at a 1-foot stump is predicted from DBH.
5. Dib along the bole is predicted from observed DBH, total height, and upper stem dob and height.
6. Observed cubic-foot volumes along the bole are calculated assuming either a neiloid or paraboloid frustrum, or cone.

FPS Taper Class System

The Forest Projection System uses species specific non-parametric look-up tables to describe tree taper profiles by region. Upper stem diameters and heights were expressed as proportions of DBH and total height, and then grouped into five taper classes (10 = open, 20 = dominant, 30 = co-dominant, 40 = intermediate, and 50 = suppressed) based on relative height. Average relative heights along the bole were calculated based on relative DBH and taper class.

Taper Class	DBH (%) --- Arney 2019 --- Region 10										
	100	90	80	70	60	50	40	30	20	10	0
10	0.00	0.04	0.10	0.23	0.37	0.51	0.63	0.73	0.83	0.92	1.00
20	0.00	0.10	0.20	0.33	0.46	0.57	0.67	0.77	0.85	0.93	1.00
30	0.00	0.17	0.30	0.44	0.55	0.65	0.74	0.81	0.88	0.94	1.00
40	0.00	0.22	0.40	0.54	0.64	0.72	0.79	0.87	0.90	0.95	1.00
50	0.00	0.28	0.50	0.64	0.73	0.80	0.85	0.90	0.93	0.97	1.00

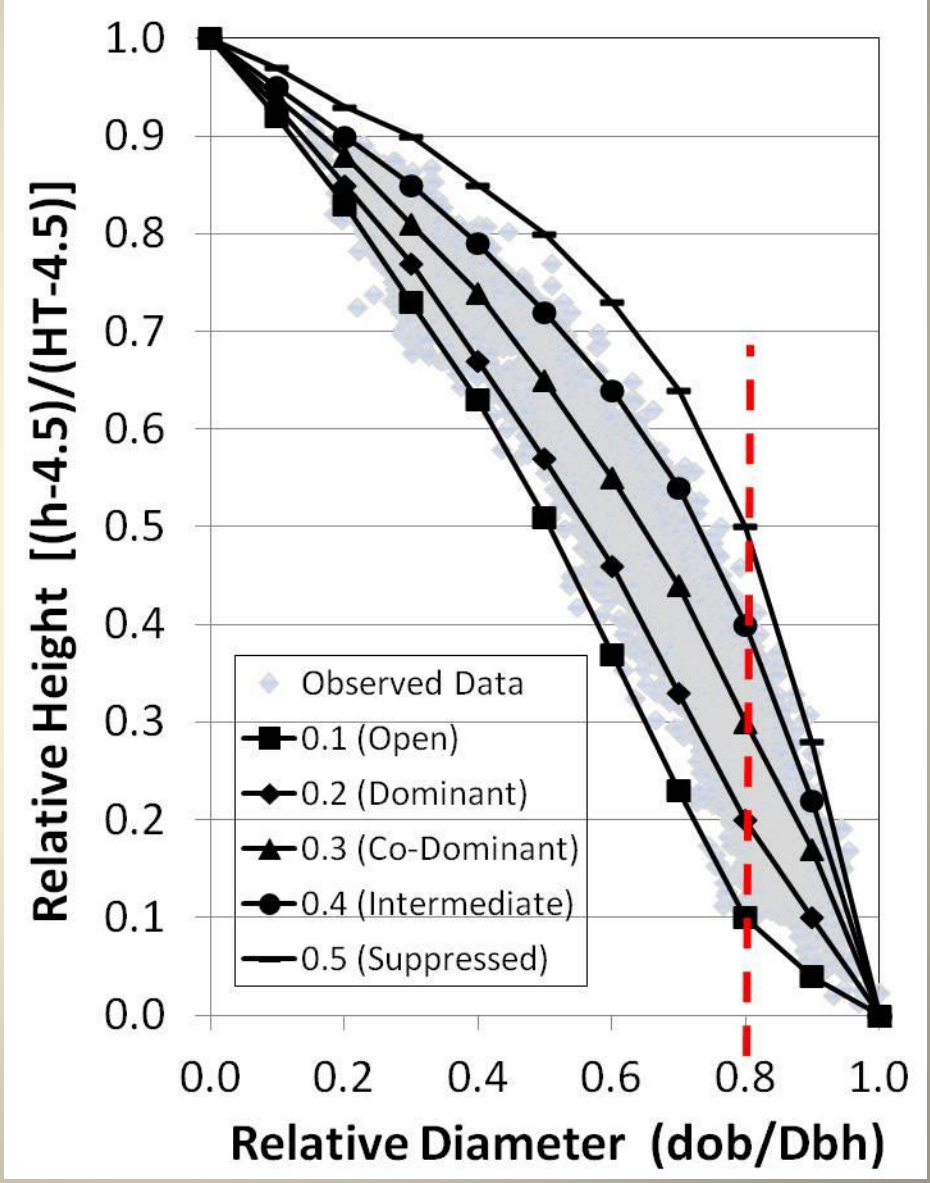
Arney, James D. 2009. Tree Taper Profiles by Species and Region. FBRI Research Paper No. 105, 20 p.

Douglas-fir Taper Profiles

Tree profiles are the average relative heights by taper class and relative DBH.

Profiles are referenced at the greatest vertical distance between curves occurring at 80% DBH.

Observed data are the IFC stem analysis data.



Using FPS Taper Class Table

Taper Class	DBH (%) --- Arney 2019 --- Region 10										
	100	90	80	70	60	50	40	30	20	10	0
10	0.00	0.04	**0.10**	0.23	0.37	0.51	0.63	0.73	0.83	0.92	1.00
20	0.00	0.10	**0.20**	0.33	0.46	0.57	0.67	0.77	0.85	0.93	1.00
30	0.00	0.17	**0.30**	0.44	0.55	0.65	0.74	0.81	0.88	0.94	1.00
40	0.00	0.22	**0.40**	0.54	0.64	0.72	0.79	0.87	0.90	0.95	1.00
50	0.00	0.28	**0.50**	0.64	0.73	0.80	0.85	0.90	0.93	0.97	1.00

For example, a tree with DBH = 8.3", total height = 42.6', height to 80% DBH = 14.1', and Taper Class = (14.1-4.5)/(42.6-4.5) = 0.253. Calculate the TC relative distance between 0.20 and 0.30 in the 80% DBH column, and use that value to interpolate between relative heights in rows labeled Taper Class 20 and 30.

Tree Attribute	Relative Diameter (% DBH)										
	100	90	80	70	60	50	40	30	20	10	0
Relative Height	0.000	0.137	0.253	0.388	0.508	0.612	0.707	0.791	0.866	0.935	1.000
Dob (in)	8.3	7.5	6.6	5.8	5.0	4.2	3.3	2.5	1.7	0.8	0.0
Height (ft)	4.5	9.7	14.1	19.3	23.8	27.8	31.4	34.6	37.5	40.1	42.6

Height = 4.5 + TC*(HT – 4.5)

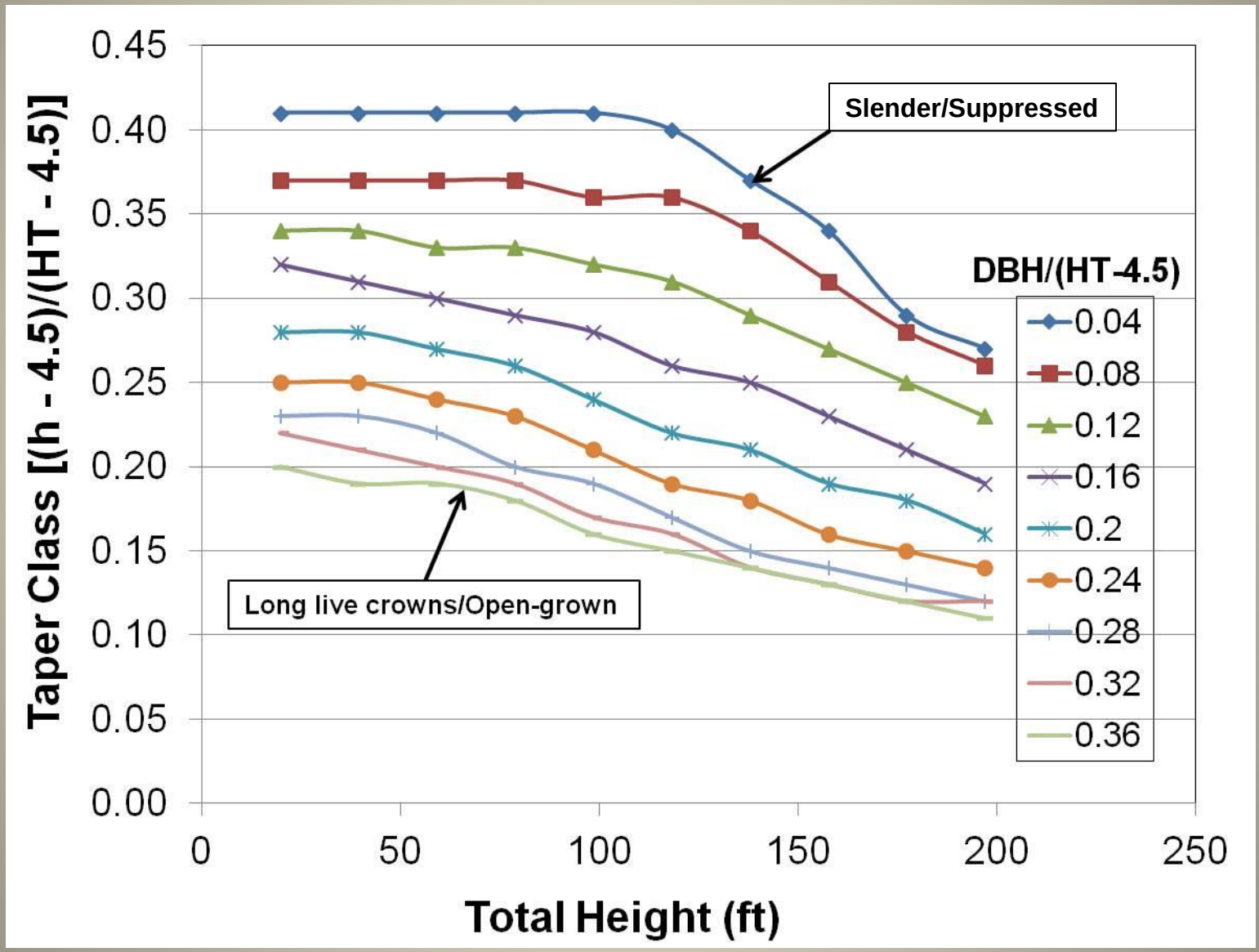
Using FPS Taper Access Table

The access table provides taper class values based on observed or predicted DBH and total height when height at 80% DBH has not been observed, and to change taper profiles over time for growth projections.

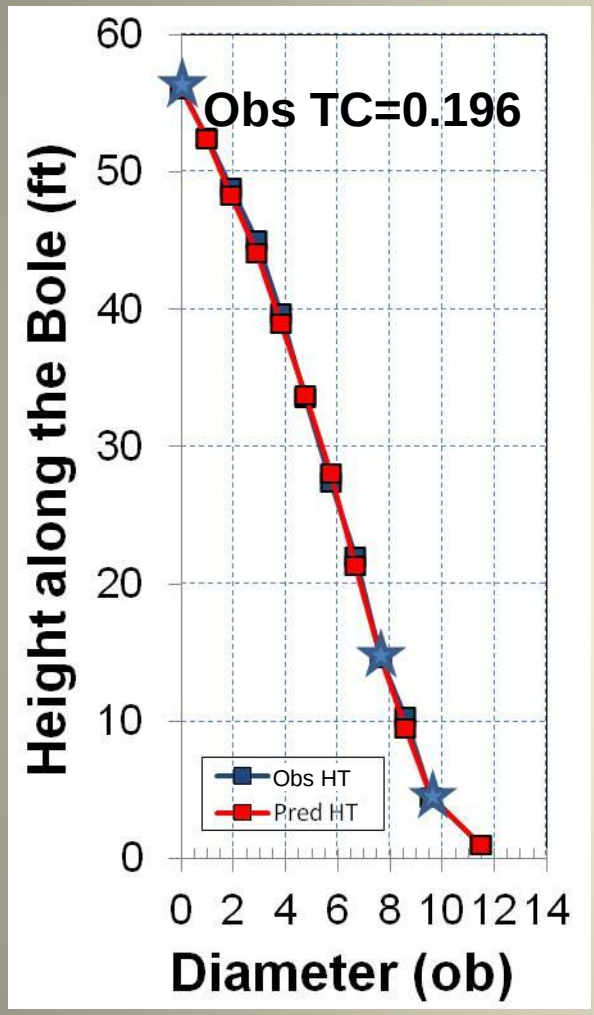
DBH/(HT-4.5)	TOTAL HEIGHT (ft) --- Arney (2019) --- Region 10									
(in/ft)	19.7	39.4	59.1	78.7	98.4	118.1	137.8	157.5	177.2	196.9
0.04	0.41	0.41	0.41	0.41	0.41	0.40	0.37	0.34	0.29	0.27
0.08	0.37	0.37	0.37	0.37	0.36	0.36	0.34	0.31	0.28	0.26
0.12	0.34	0.34	0.33	0.33	0.32	0.31	0.29	0.27	0.25	0.23
0.16	0.32	0.31	0.30	0.29	0.28	0.26	0.25	0.23	0.21	0.19
0.20	0.28	0.28	0.27	0.26	0.24	0.22	0.21	0.19	0.18	0.16
0.24	0.25	0.25	0.24	0.23	0.21	0.19	0.18	0.16	0.15	0.14
0.28	0.23	0.23	0.22	0.20	0.19	0.17	0.15	0.14	0.13	0.12
0.32	0.22	0.21	0.20	0.19	0.17	0.16	0.14	0.13	0.12	0.12
0.36	0.20	0.19	0.19	0.18	0.16	0.15	0.14	0.13	0.12	0.11

For example, a tree with DBH = 8.3" and total height = 42.6', and DBH/(HT-4.5) = 0.218 has an estimated Taper Class = 0.261 using double linear interpolation.

FPS Taper Access Curves



FPS Taper Class – Example



Observed volume = 11.58 ft^3
Estimated volume = 11.44 ft^3

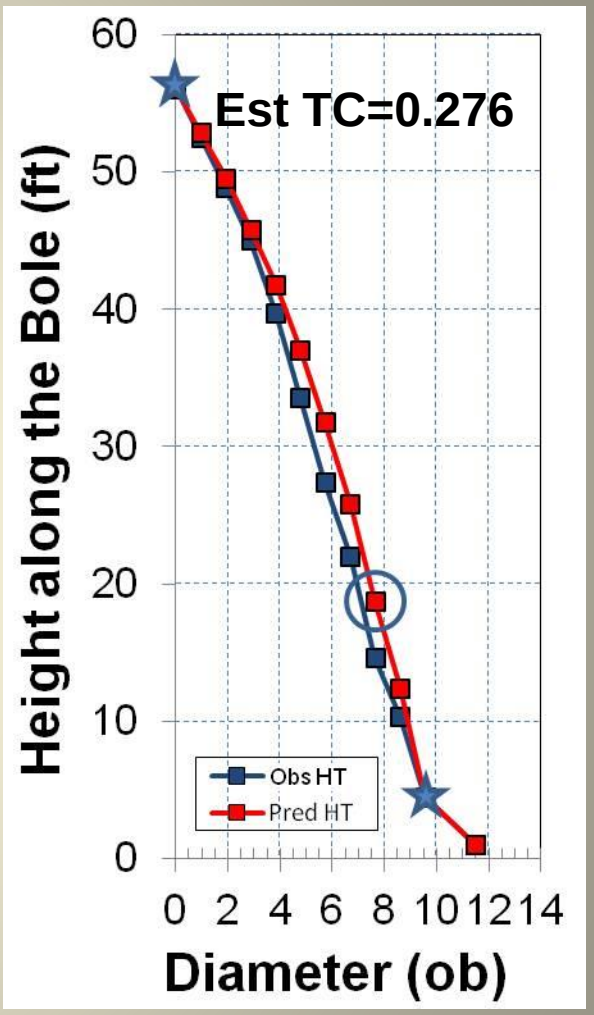
Observed Tree Data ★
DBH = 9.6 in
Total Height = 56.1 ft
Ht at 80% DBH = 14.6 ft

Observed TC

$$TC = \frac{(14.6 - 4.5)}{(56.1 - 4.5)} = 0.196$$

Predicted Ht for TC = 0.276

Ht = 4.5 +0.276*(56.1 - 4.5)
= 18.7 ft

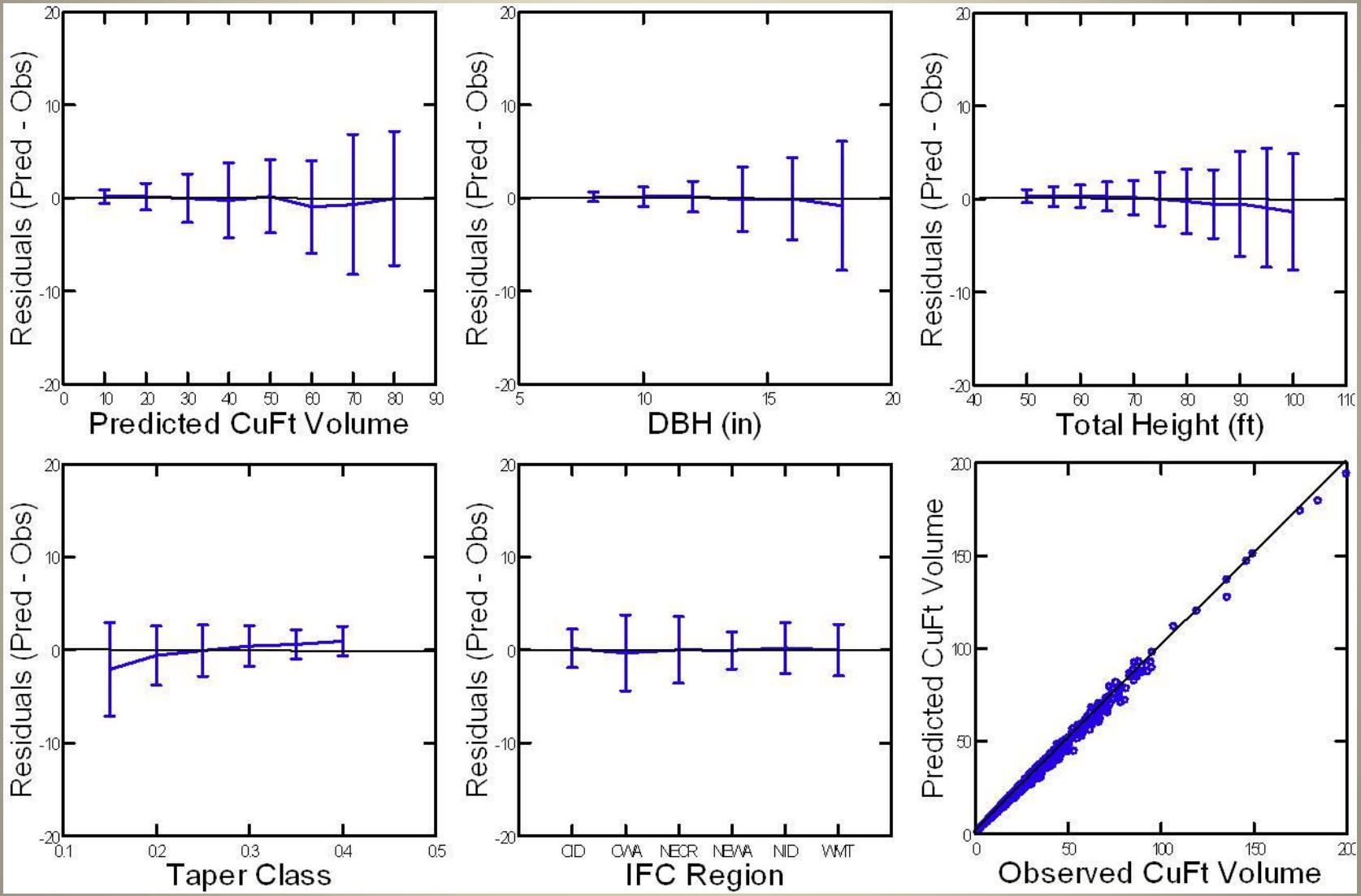


Observed volume = 11.58 ft^3
Estimated volume = 12.92 ft^3

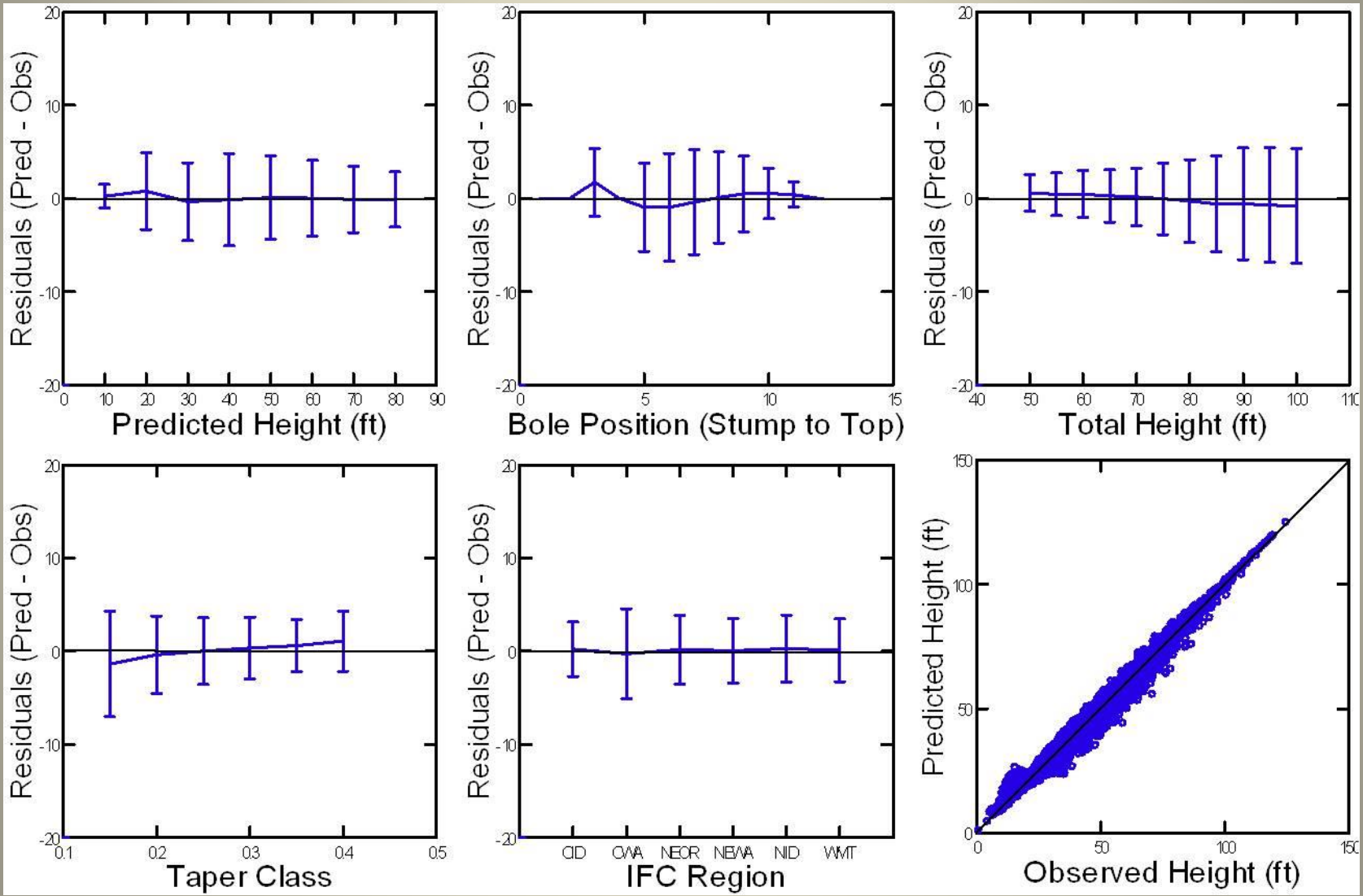
FPS Taper Class – Residual Analysis

Statistic (Residual = Pred - Obs)	Observed Taper Class		Estimated Taper Class	
	Total ft^3 Volume	Height (ft)	Total ft^3 Volume	Height (ft)
n	1,085	13,020	1,085	13,020
Mean	0.000	0.088	-0.231	-0.038
Standard Dev	1.513	1.929	2.923	2.449
Standard Error	0.046	0.017	0.089	0.021
Root Mean Sq Error	1.517	6.950	2.931	2.449
Mean % Resid	0.553	1.172	0.531	1.411
Mean Abs Resid	0.977	1.120	1.687	1.527
Minimum	-8.737	-15.214	-31.238	-16.918
Maximum	7.070	10.678	11.445	13.730
Low Value -- 95% CI	-0.090	0.055	-0.405	-0.080
High Value -- 95% CI	0.090	0.121	-0.057	0.005

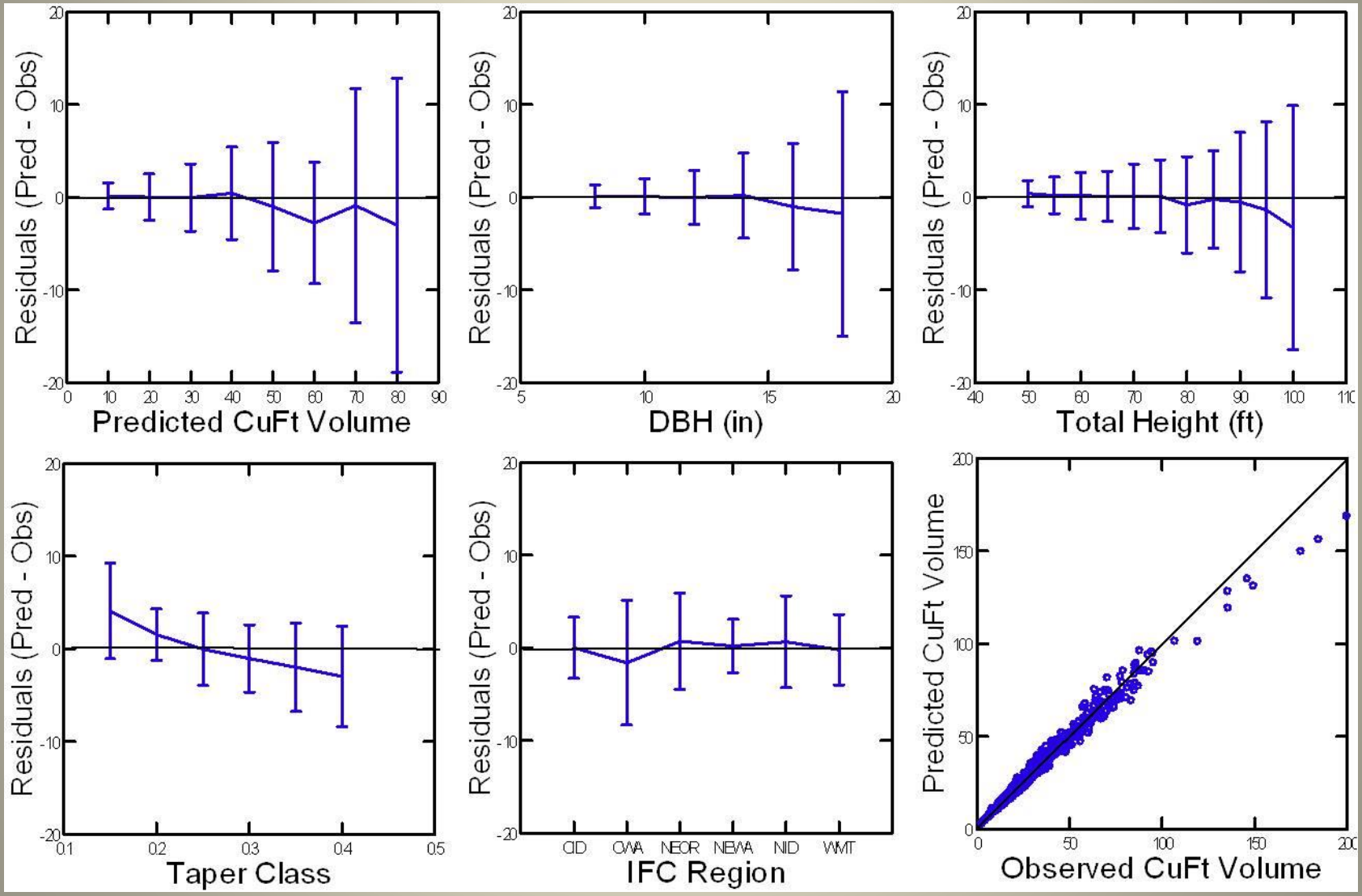
Volume Residuals – Observed TC



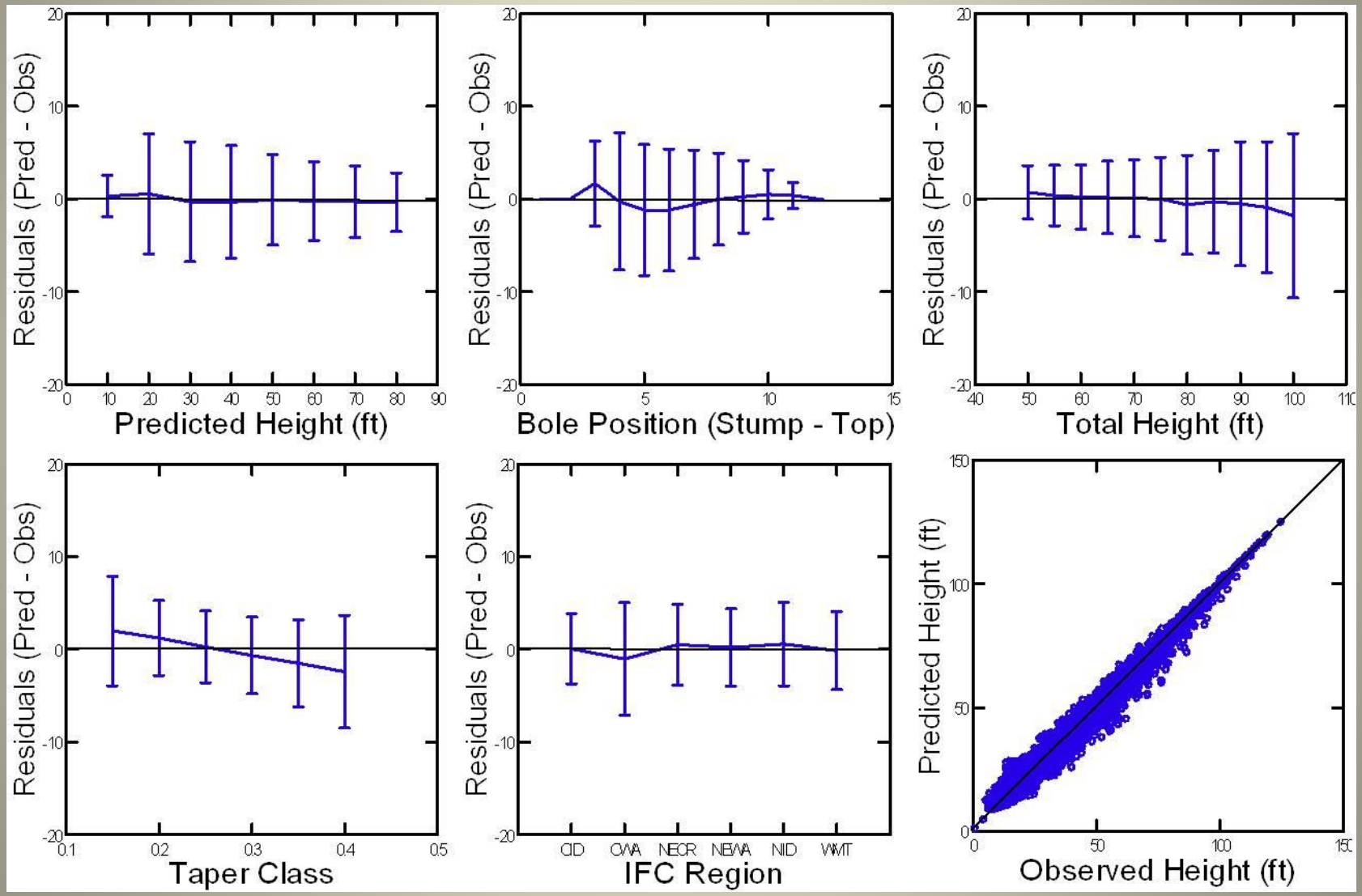
Height Residuals – Observed TC



Volume Residuals – Estimated TC



Height Residuals – Estimated TC



Flewelling 2 & 3-Point Taper System

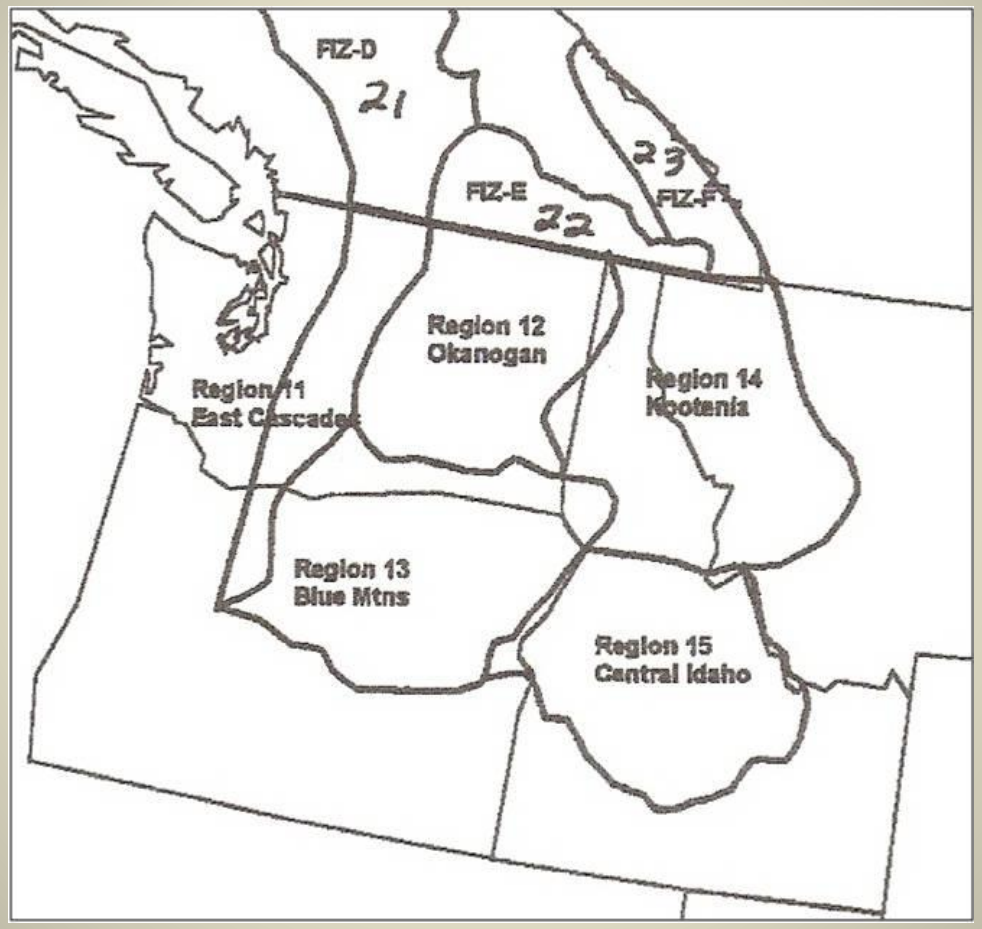
The Flewelling taper system was developed for eastern Oregon, eastern Washington, Idaho, and western Montana as an INGY cooperative project. The system inputs include either DBH and total height (2-point), or DBH, total height and an upper stem diameter/height pair (3-point). The 3-point system, which passes through all measured stem data, uses upper stem data to localize predicted 2-point stem-profiles.

Flewelling, J.W. 1996. Development of Inland Growth & Yield Cooperative Taper Models. Internal Report. INGY Cooperative. College of Forestry, University of Montana.

Flewelling, J.W. and L.M. Raynes. 1993. Variable-shape stem-profile predictions for western hemlock. Part I. Predictions from DBH and total height. Can. J. For. Res. 23: 520-536.

Flewelling, J.W. 1993. Variable-shape stem-profile predictions for western hemlock. Part II. Predictions from DBH, total height, and upper stem measurements. Can. J. For. Res. 23: 537-544.

INGY Taper Regions



National Volume Estimator Library

The Volume Estimator Library is a collection of volume and taper equations used by the USDA Forest Service which are also available to the public.

EXCEL add-in functions were created for Flewelling's taper system and added to the volume DLL library.

The add-in functions are linked to EXCEL spreadsheets to calculate volumes and other tree attributes.

USDA Regions	INGY Regions	INGY Codes	IFC Regions	Flewelling 2-Point	Flewelling 3-Point
6	East Cascades	11	CWA	I11FW2W202	I00FW3W202
	Okanogan	12	NEWA	I12FW2W202	I00FW3W202
	Blue Mountains	13	NEOR	I13FW2W202	I00FW3W202
1	Kootenai	14	NID / WMT	I00FW2W202	I00FW3W202
4	Central Idaho	15	CID	I15FW2W202	I15FW3W202

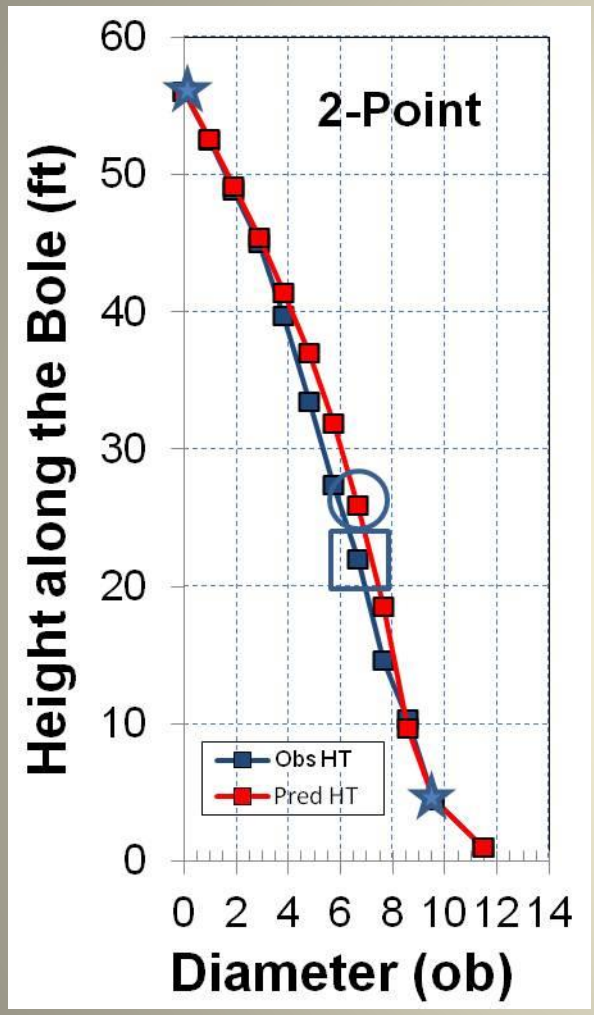
National Volume Estimator Library (NVEL)
https://www.fs.fed.us/forestmanagement/products/measurement/volume/nvel/index.php

EXCEL Volume Library Functions

EXCEL Add-In NVEL Functions			
CalcHt (2-Point)		GetXHtAdv (3-Point)	
Variable	Input	Variable	Input
Region	1	Region	1
Forest	0	Forest	0
VolEquNum	"I15FW2W202"	VolEquNum	"I15FW3W202"
DBH	9.55	DBH	9.55
TotalHt	56.1	TotalHt	56.1
UpStemDia (ib)	6.25	MerchHt	0
Pred Height	**25.9**	DIB	6.25
		LogOrFeet	0
		LogLength	0
		UpStemHT	14.6
		UpStemDia (ib)	7.09
		AvgZ	0
		RefHt	0
		FormClass	0
		Product	0
		DblBarkBH	0
		BarkRatio	0
		BasalArea	0
		SiteIndex	0
		Pred Height	**23.3**

Add-in functions for 2-point and 3-point taper models were solved for heights associated with upper stem DIBs, which were estimated from DOBs using predicted double bark thickness.

Flewelling – Example



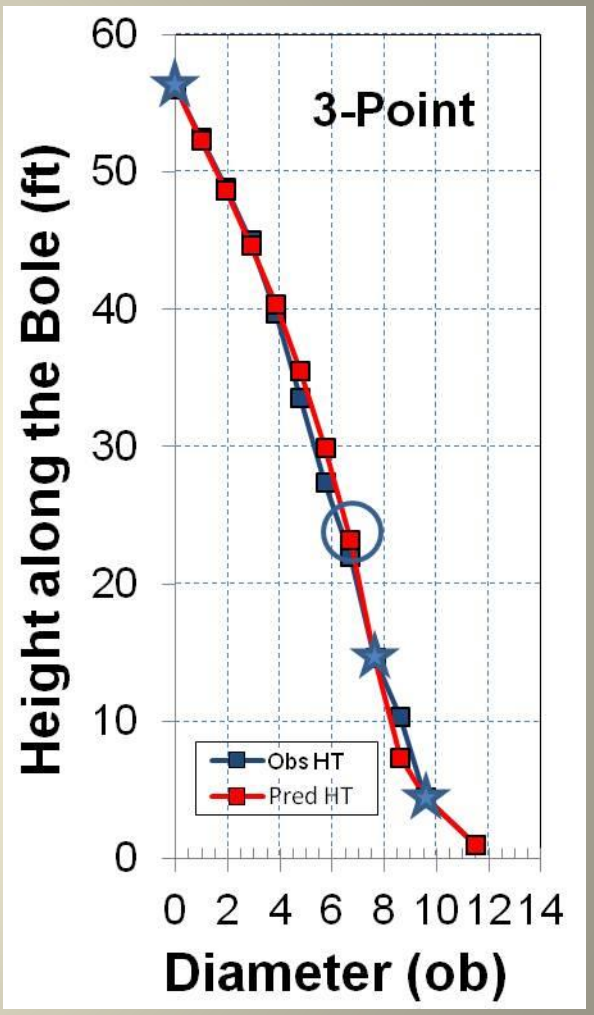
Observed volume = 11.58 ft^3
Estimated volume = 12.63 ft^3

Observed Tree Data ★
DBH = 9.6 in
Total Height = 56.1 ft
Ht at 80% DBH = 14.6 ft

Taper Class = $\frac{14.6 - 4.5}{56.1 - 4.5}$
= 0.196

○ = predicted height at 6.25" dib or 6.69" dob

□ = observed height at 6.25" dib or 6.69" dob

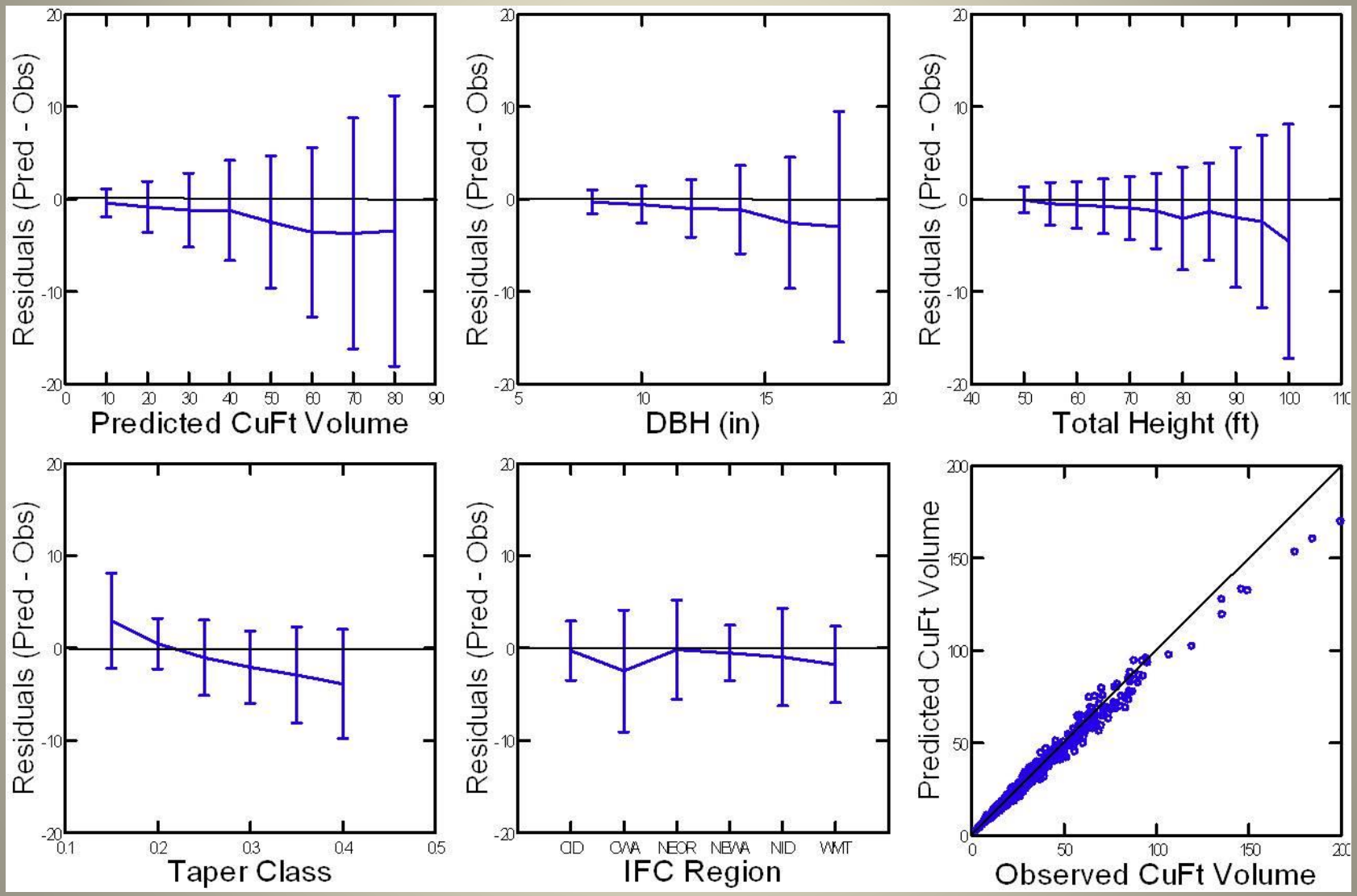


Observed volume = 11.58 ft^3
Estimated volume = 11.65 ft^3

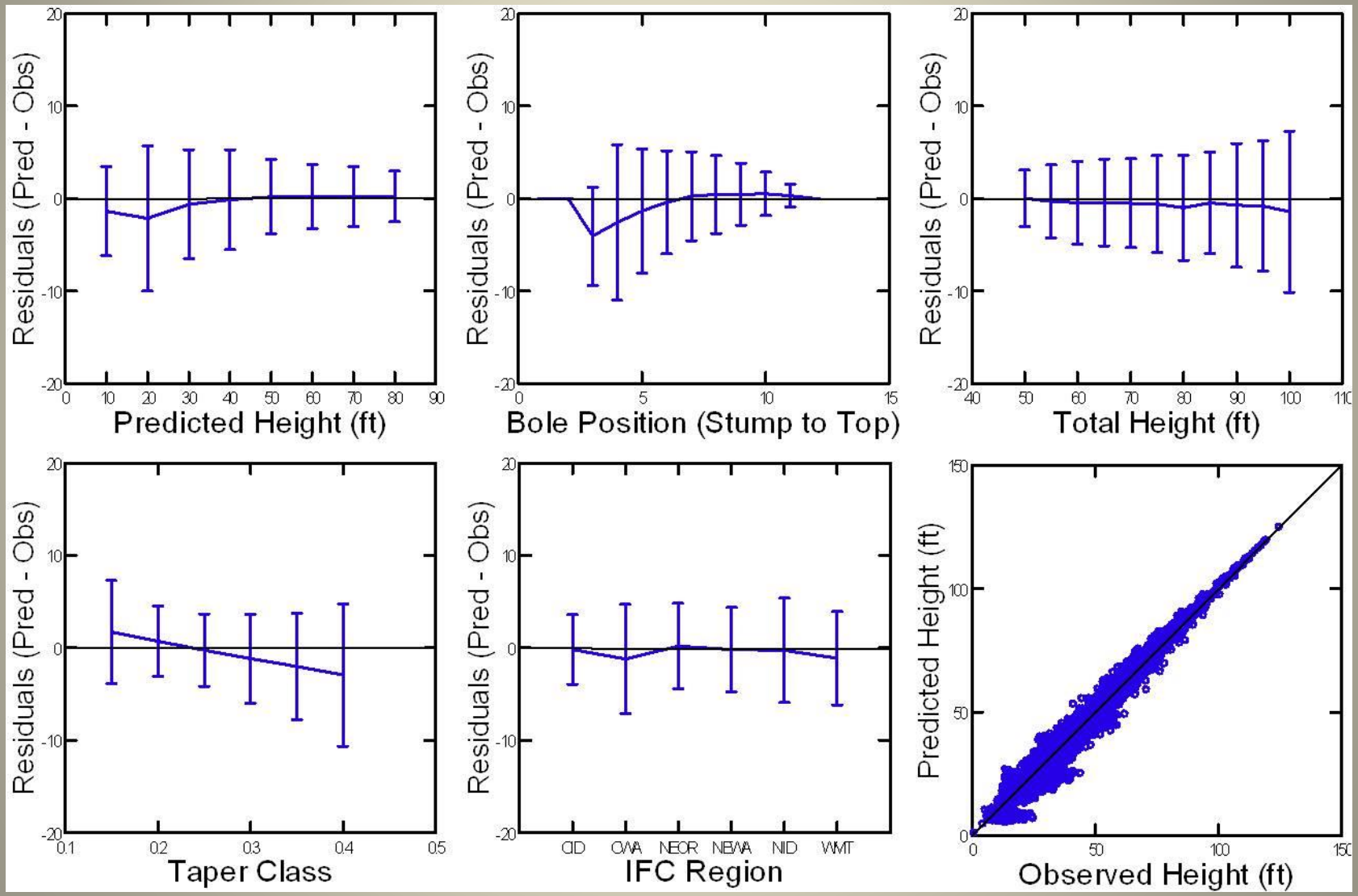
Flewelling – Residual Analysis

Statistic (Residual = Pred - Obs)	2-Point (DBH,HT)		3-Point (DBH,HT,TC)	
	Total ft^3 Volume	Height (ft)	Total ft^3 Volume	Height (ft)
n	1,085	13,020	1,085	13,020
Mean	-1.232	-0.534	-0.175	0.183
Standard Dev	2.879	2.634	1.203	1.875
Standard Error	0.087	0.023	0.037	0.016
Root Mean Sq Error	3.130	2.687	1.215	1.884
Mean % Resid	-3.478	-3.156	-0.427	-1.258
Mean Abs Resid	1.932	1.626	0.721	1.138
Minimum	-30.136	-19.136	-10.339	-16.031
Maximum	10.822	12.866	8.326	9.358
Low Value -- 95% CI	-1.403	-0.579	-0.247	0.150
High Value -- 95% CI	-1.060	-0.489	-0.104	0.215

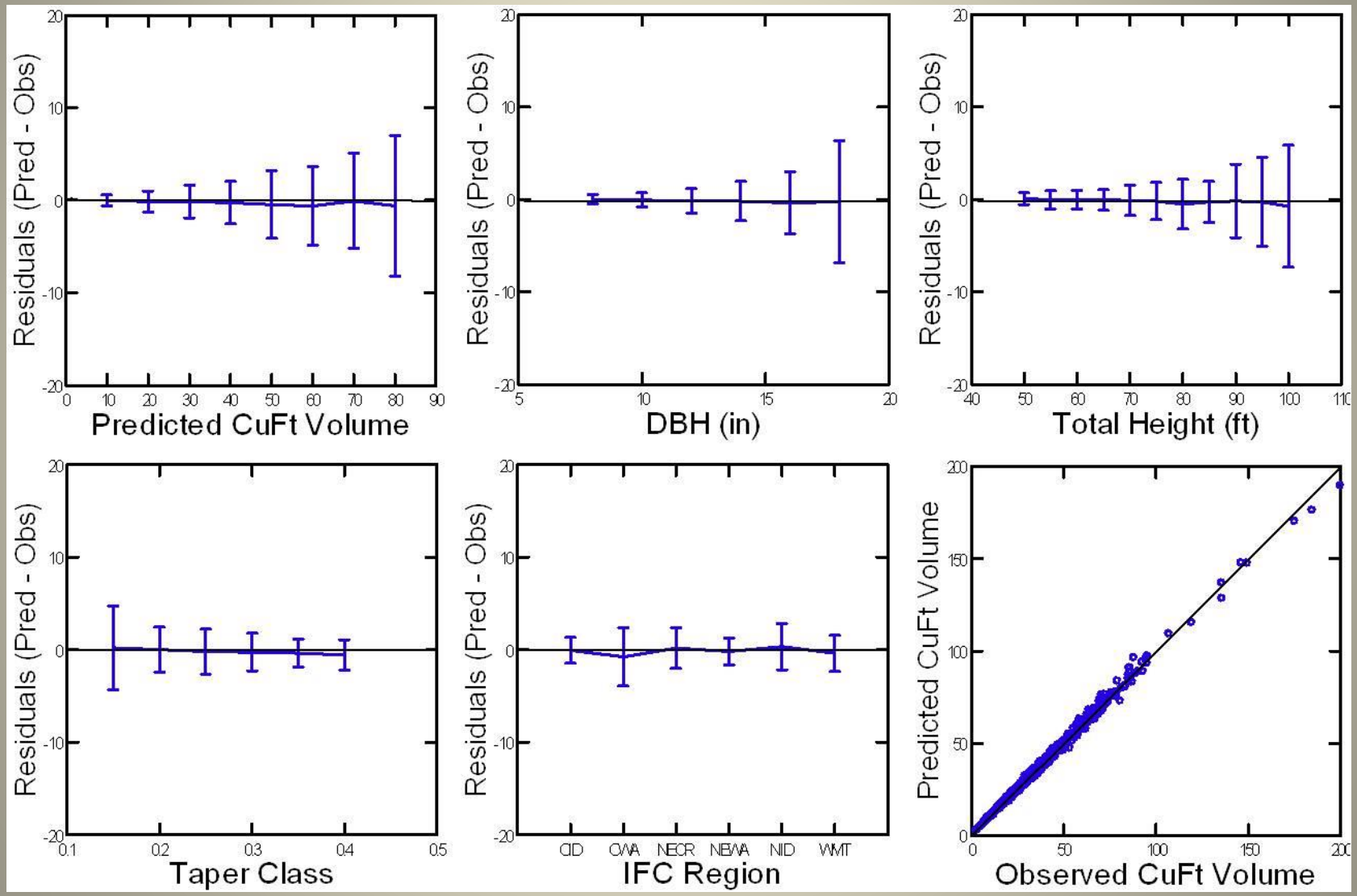
Volume Residuals – 2-Point



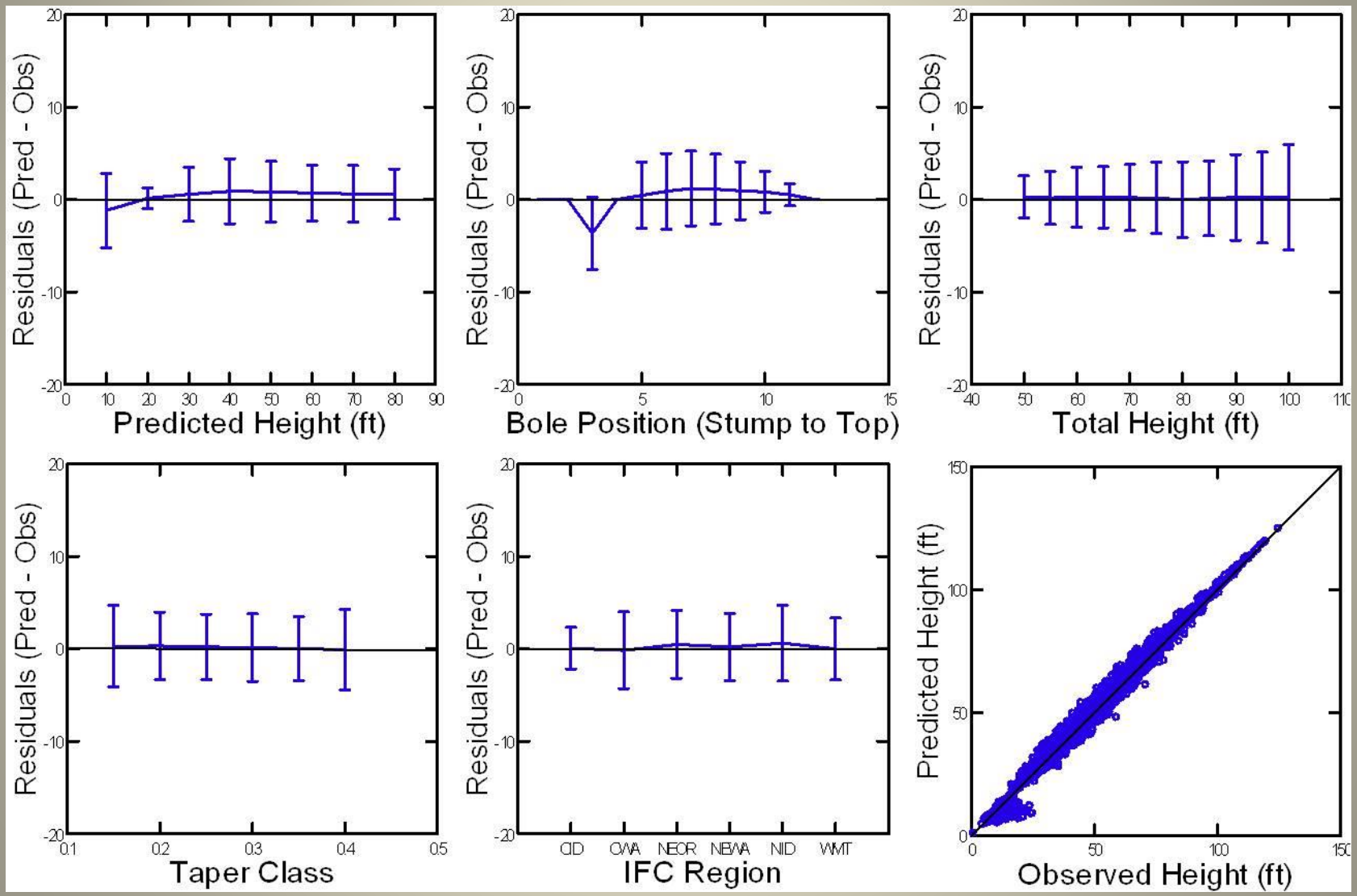
Height Residuals – 2-Point



Volume Residuals – 3-Point



Height Residuals – 3-Point



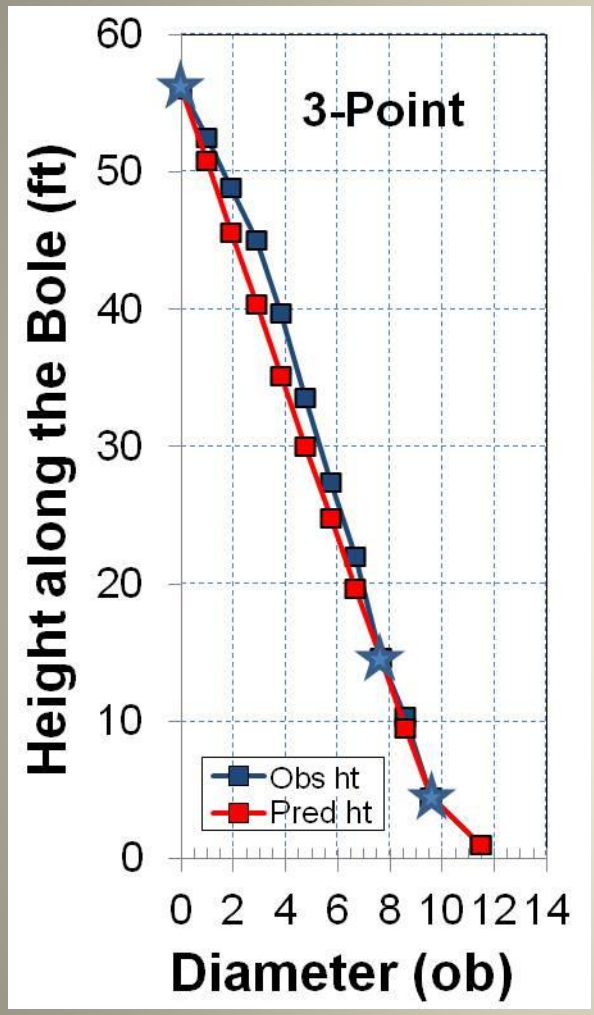
Natural Cubic Spline Interpolation

Natural cubic splines are smooth, continuous functions that pass through all observed diameter/height pairs along the bole. They consist of a series of third-order polynomials fit together in a piecewise fashion, under the condition that the function is continuous and differentiable at the observed data points. In addition, the second derivative of the splines at end points (i.e., at tree base = (DBH,4.5) and tree top = (0,total height) are zero.

Given observed dob/height data points along the bole, a computer program fits a series of piece-wise cubic polynomials between each of the data points resulting in a smooth continuous curve. The piece-wise spline function is used to interpolate heights for observed dob values.

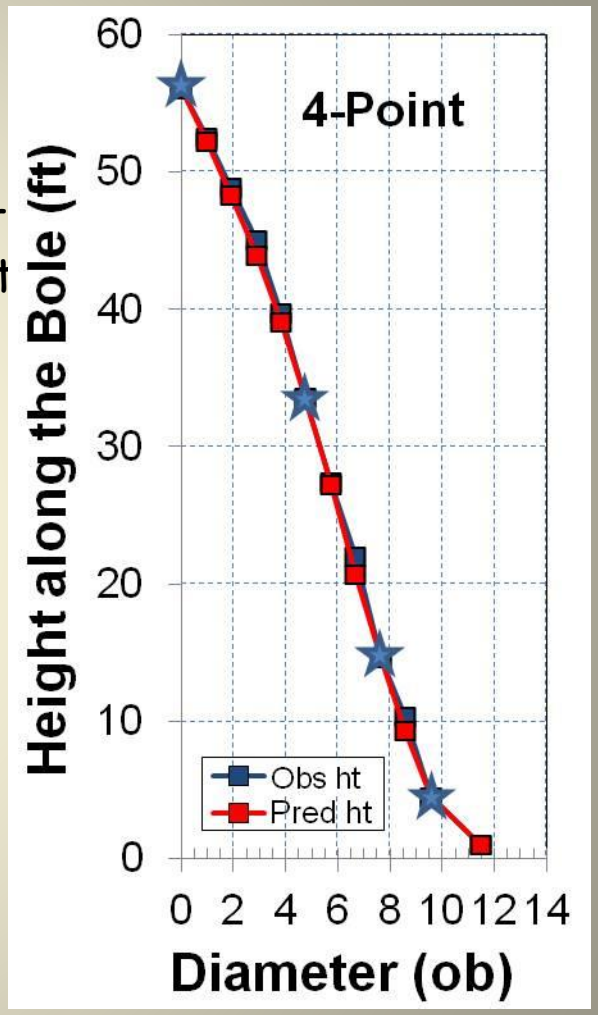
Press, W.H., B.P. Flannery, S.A. Teukolsky, and W.T. Vetterling. 1989. Numerical Recipes – The Art of Scientific Computing (FORTRAN Version). Cambridge University Press, New York, 702 p.

Cubic Spline – Example



Observed volume = 11.58 ft^3
Estimated volume = 10.61 ft^3

Observed Tree Data ★
DBH = 9.6 in
Total Height = 56.1 ft
Ht at 80% DBH = 14.6 ft
Ht at 50% DBH = 33.5 ft

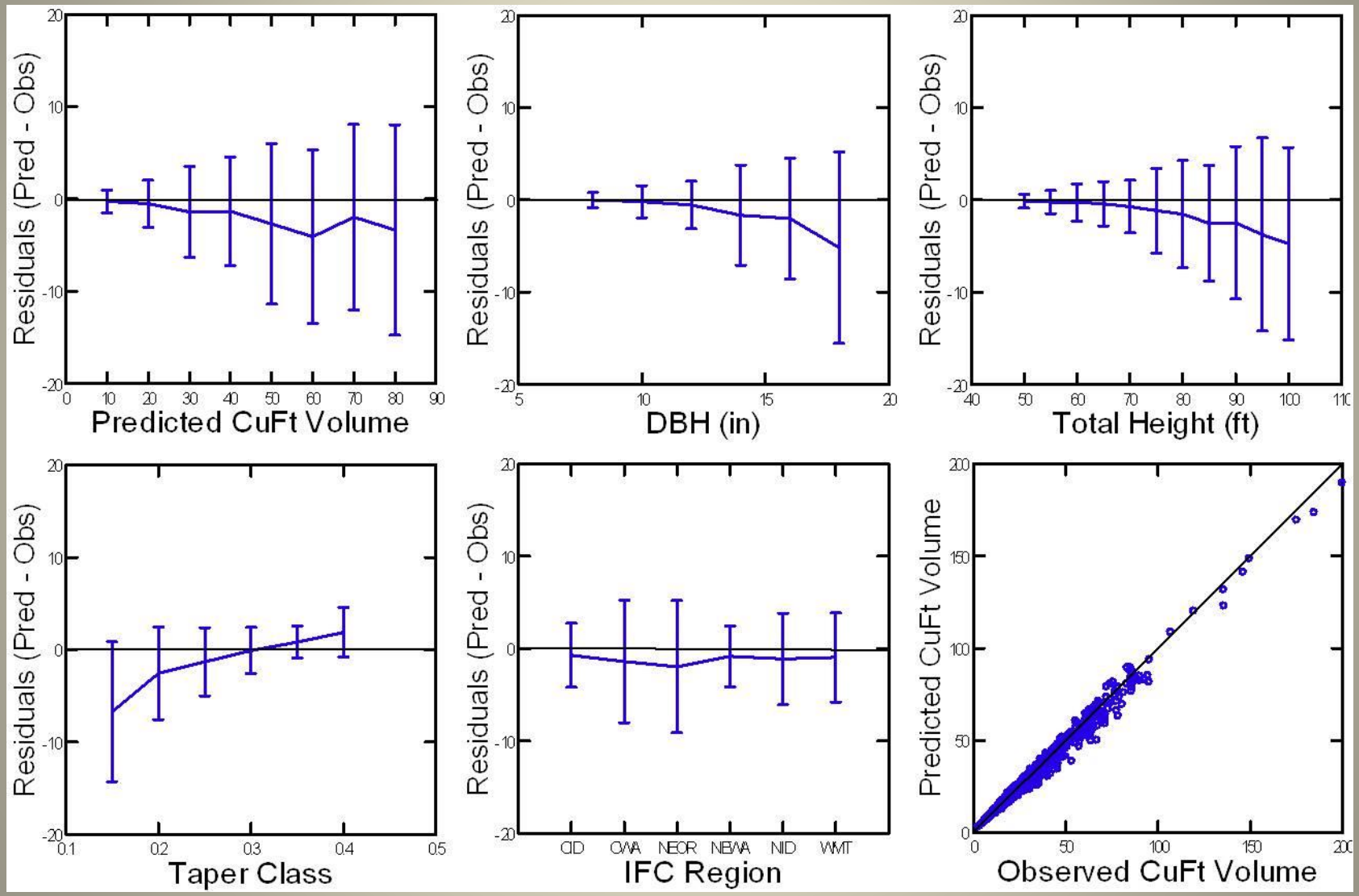


Observed volume = 11.58 ft^3
Estimated volume = 11.31 ft^3

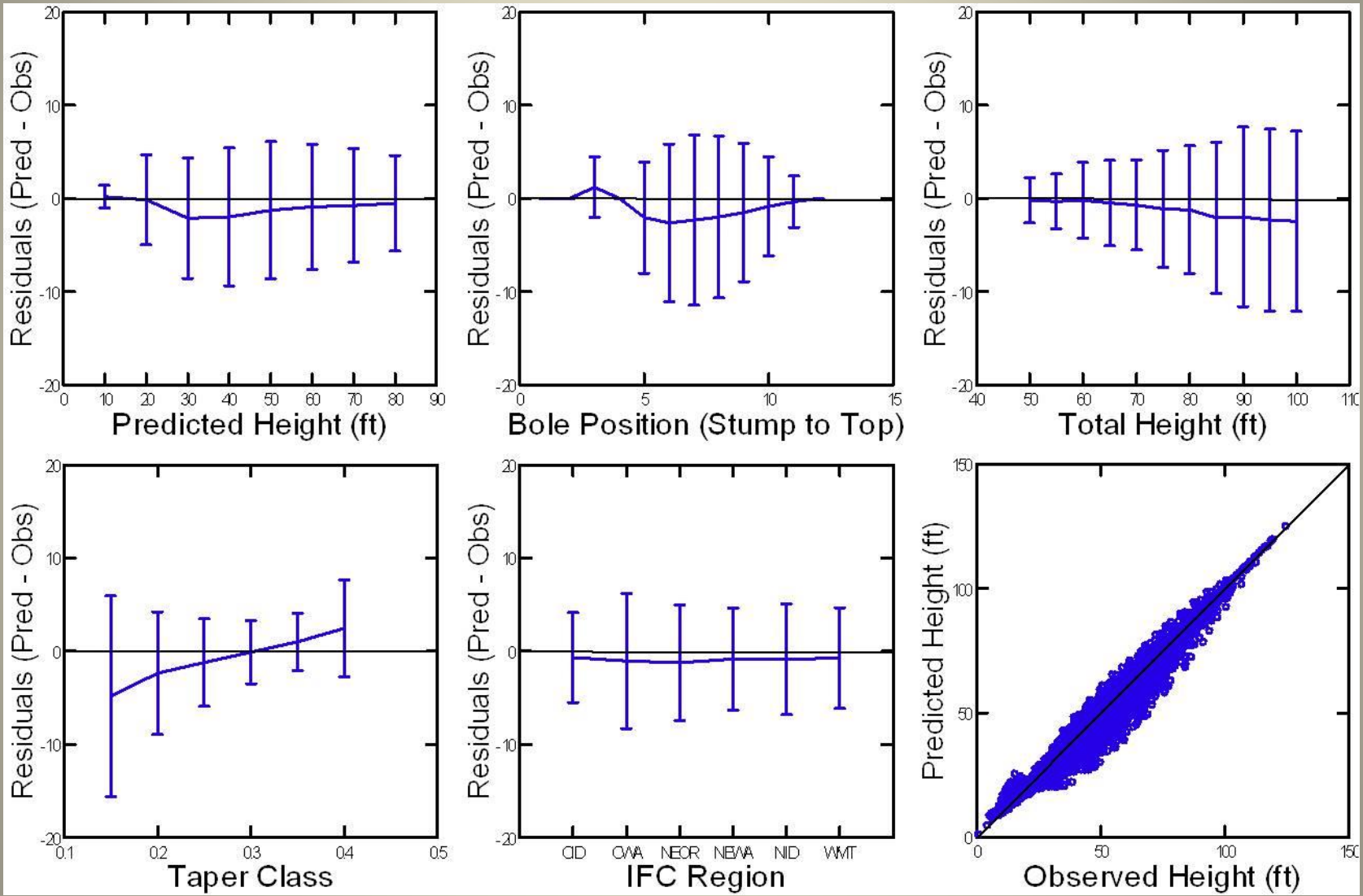
Cubic Spline – Residual Analysis

Statistic (Residual = Pred - Obs)	Spline – 3-Point (DBH,HT,TC=80)		Spline – 4-Point (DBH,HT,TC=80/50)	
	Total ft^3 Volume	Height (ft)	Total ft^3 Volume	Height (ft)
n	1,085	13,020	1,085	13,020
Mean	-1.097	-0.880	0.029	0.142
Standard Dev	2.614	3.063	0.454	1.254
Standard Error	0.079	0.027	0.014	0.011
Root Mean Sq Error	2.833	3.187	0.455	1.262
Mean % Resid	-2.889	-1.214	0.029	0.567
Mean Abs Resid	1.669	1.756	0.267	0.674
Minimum	-16.972	-21.023	-4.033	-12.667
Maximum	6.618	10.232	2.469	10.075
Low Value -- 95% CI	-1.252	-0.933	0.002	0.120
High Value -- 95% CI	-0.941	-0.828	0.056	0.163

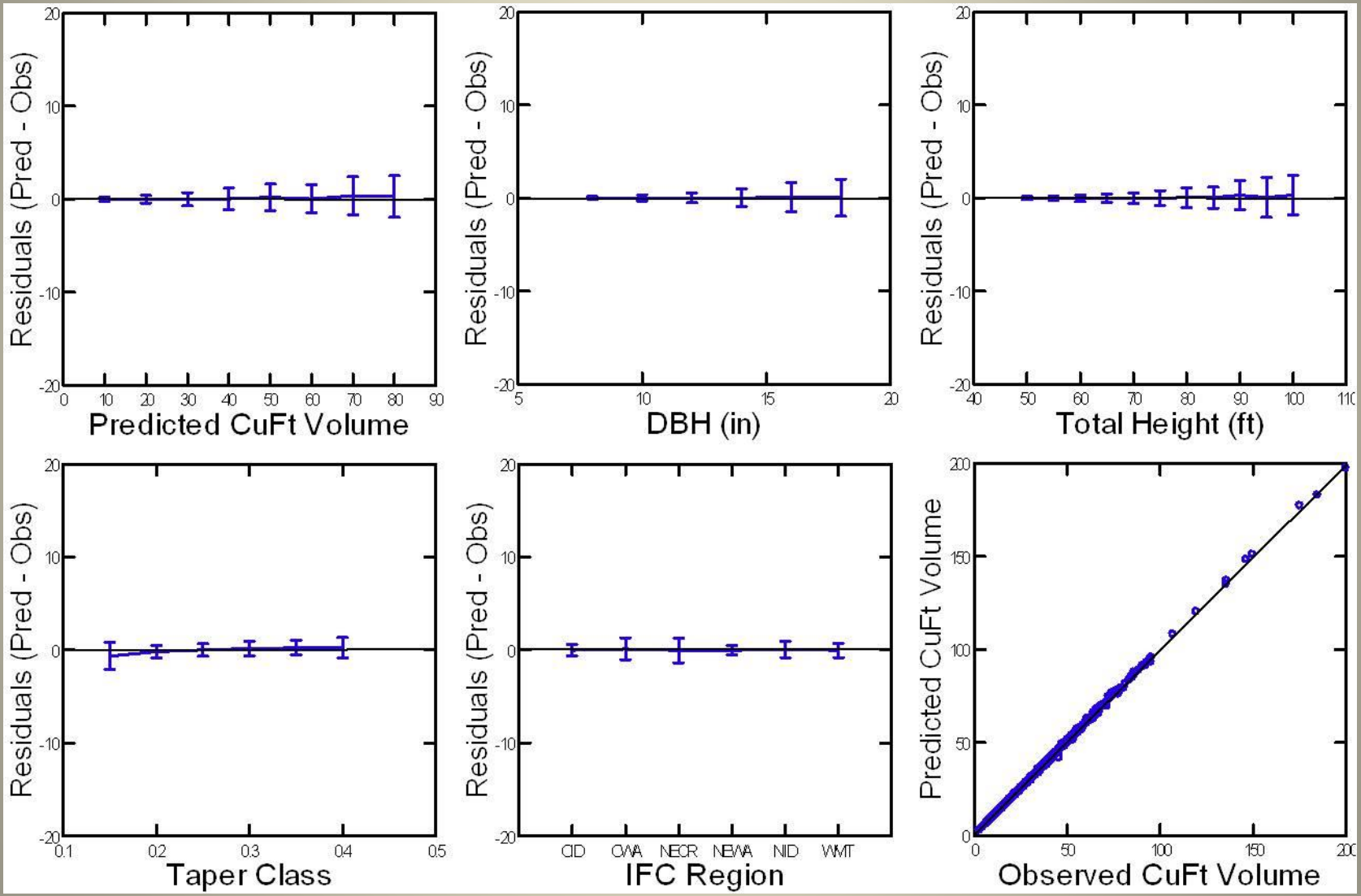
Volume Residuals – 3-Point



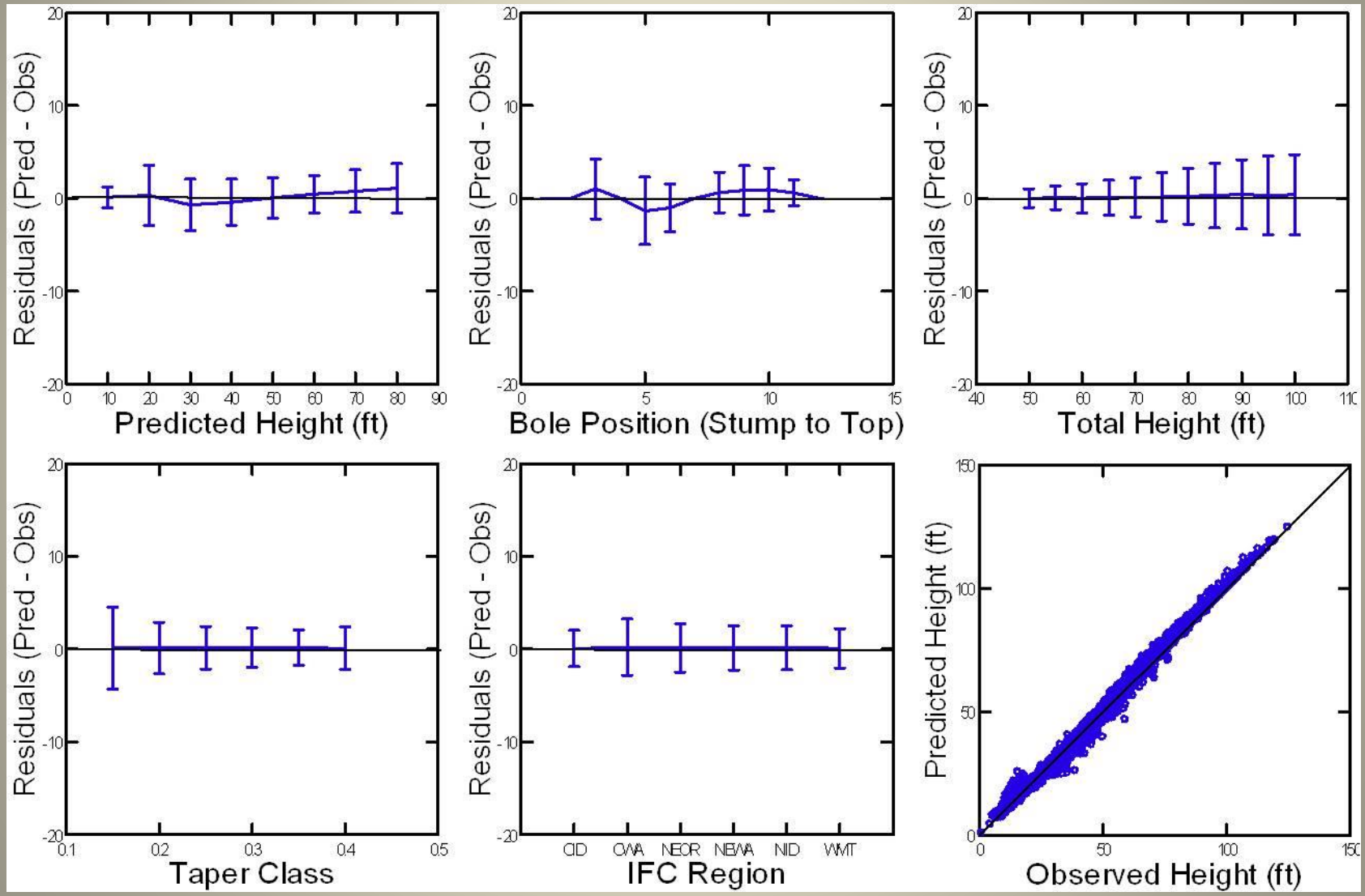
Height Residuals – 3-Point



Volume Residuals – 4-Point



Height Residuals - 4-Point



Summary

Taper System	Input	Residual Statistics					
		Mean		Std Error		RMSE	
		Volume (ft^3)	Height (ft)	Volume (ft^3)	Height (ft)	Volume (ft^3)	Height (ft)
FPS Taper Class	Obs TC	0.000	0.088	0.046	0.017	1.517	6.950
	Est TC	-0.231	-0.038	0.089	0.021	2.931	2.449
Flewelling	2-PT	-1.232	-0.534	0.087	0.023	3.130	2.687
	3-PT	-0.175	0.183	0.037	0.016	1.215	1.884
Spline	3-PT	-1.097	-0.880	0.079	0.027	2.833	3.187
	4-PT	0.029	1.254	0.014	0.011	0.455	1.262