Section 1.1

This lesson will cover Section 1.1 in your e-Text. Work through each of the following tasks, carefully filling out the following pages.

Section 1.1 An Introduction to Angles: Degree and Radian Measure

- Work through TTK #1 then do problem #1
- Work through Objective 1 then do problems #2-3
- Work through Objective 2 then do problems #4-5
- Work through Objective 3 then do problems #6-9
- Work through Objective 4 then do problems #10-14
- Work through Objective 5 then do problems #15-16

Section 1.1 an Introduction to Angles: Degree and Radian Measure

1.1 Things To Know

1. Sketching the Graph of a Circle

Can you sketch the graph of a circle? Try working through a “You Try It” problem or watch the video.

NOW WORK SECTION 1.1 HW EXERCISE #1
Section 1.1 Introduction

What is the definition of a vertex?

What is the definition of the initial side?

What is the definition of the terminal side?

Sketch an angle with positive measure, labeling the vertex, initial side, and terminal side. Do the same for an angle with negative measure.

What does it mean for an angle to be in standard position?

Sketch an angle in standard position having positive measure.

Sketch an angle in standard position having negative measure.
Section 1.1 Objective 1 Understanding Degree Measure

In the degree measure system, what is the symbol used to indicate a degree? How many degrees are in a one complete counterclockwise rotation?

Sketch three coordinate planes, illustrating angles of 360, 90, and -45 degrees respectively. (See Figures 3, 4, and 5.)

What is the definition of an acute angle?

What is the definition of an obtuse angle?

What is the definition of a quadrantal angle?

What is the term for an angle of exactly 90 degrees?

What is the term for an angle of exactly 180 degrees?

What does it mean for angles to be coterminal?

Sketch the two coordinate planes illustrating common positive and negative angles as seen in Figure 6.
Work through the video accompanying Example 1 showing all work below. Draw each angle in standard position and state the quadrant in which the terminal side of the angle lies or the axis on which the terminal side of the angle lies.

a. \( \theta = 60^\circ \)  
b. \( \alpha = -270^\circ \)  
c. \( \beta = 420^\circ \)

NOW WORK SECTION 1.1 HW EXERCISES #2-3

Section 1.1 Objective 2 Finding Coterminal Angles Using Degree Measure

What is the definition of Coterminal Angles?

Starting with a given angle, how can you obtain coterminal angles? (See the coterminal angle definition box.)

What notation is used to denote the angle of least nonnegative measure that is coterminal with?
Work through the video with Example 2 and show all work below.
Find the angle of least nonnegative measure, $\theta_c$, that is coterminal with $\theta = -697^\circ$.

**NOW WORK SECTION 1.1 HW EXERCISES #4-5**

Section 1.1 Objective 3 Understanding Radian Measure

Carefully work through the animation seen next to Objective 3 on page 1.1-13 and answer the questions below:

Draw a circle centered at the origin having a radius of $r$ units. What is the equation of the circle?

What is the definition of a central angle?

What is the definition of an intercepted arc? What variable is typically used to represent it?

On the graph of your circle, above, draw a central angle so that the intercepted arc is the same length as the radius of your circle.

What is the measure of this central angle called?
What is the definition of a **radian**?

Approximately how many radians are there in a circle?

Carefully work through the animation seen near the bottom of page 1.1-13 and answer the questions below:

What is the formula for the circumference of a circle of radius $r$ units?

(Fill in the blank) A central angle of $360^\circ$ intercepts an arc length of __________.

Complete the proportion below as seen in the animation.

\[
\frac{360^\circ}{r} = \boxed{\text{__________}}
\]

Fill in the box: $360^\circ = \boxed{\text{__________}}$ radians.

Fill in the box: $180^\circ = \boxed{\text{__________}}$ radians.
Sketch three coordinate planes, illustrating angles of $2\pi$, $\frac{\pi}{2}$, and $\frac{-\pi}{4}$ radians respectively.
(See Figures 10, 11, and 12.)

Sketch two coordinate planes illustrating common positive and negative angles in radians as seen in Figure 13.

Work through the interactive video accompanying Example 3 showing all work below. Draw each angle in standard position and state the quadrant in which the terminal side of the angle lies or the axis on which the terminal side of the angle lies.

\begin{align*}
a. \quad & \theta = \frac{\pi}{3} \\
b. \quad & \alpha = -\frac{3\pi}{2} \\
c. \quad & \beta = \frac{7\pi}{3}
\end{align*}

NOW WORK SECTION 1.1 HW EXERCISES #6-9
Section 1.1 Objective 4 Converting between Degree Measure and Radian Measure

To convert degrees to radians, multiply by _______________________.

To convert radians to degrees, multiply by _______________________.

Work through the interactive video with Example 4 and show all work below.
Convert each angle given in degree measure into radians.

a. $45^\circ$  
b. $-150^\circ$  
c. $56^\circ$

NOW WORK SECTION 1.1 HW EXERCISES #10-12

Work through the interactive video with Example 5 and show all work below.
Convert each angle given in radian measure into degrees. Round to two decimal places if needed.

a. $\frac{2\pi}{3}$ radians  
b. $-\frac{11\pi}{6}$ radians  
c. 3 radians

NOW WORK SECTION 1.1 HW EXERCISES #13-14
Section 1.1 Objective 5 Finding Coterminal Angles Using Radian Measure

For any angle $\theta$ and for any nonzero integer $k$, we can find a coterminal angle using what expression?

Work through Example 6 and show all work below.

Find three angles that are coterminal with $\theta = \frac{\pi}{3}$ using $k = 1$, $k = -1$, and $k = -2$.

Work through the video with Example 7 and show all work below.

Find the angle of least nonnegative measure, $\theta_c$, that is coterminal with $\theta = -\frac{21\pi}{4}$.

NOW WORK SECTION 1.1 HW EXERCISES #15-16
Section 1.3

This Lesson will cover Section 1.3 in your e-Text. Work through each of the following tasks, carefully filling out the following pages.

Section 1.3 Triangles

- Work through Objective 1 then do problems #1-2
- Work through Objective 2 then do problems #3-4
- Work through Objective 3 then do problems #5-8
- Work through Objective 4 then do problems #9-14
- Work through Objective 5 then do problems #15-16

Section 1.3 Triangles

Section 1.3 Objective 1 Classifying Triangles

What does it mean for two angles or sides of a triangle to be congruent?

What is an acute triangle?

What is an obtuse triangle?

What is a right triangle?

Sketch and label an acute, obtuse, and right triangle, as seen in Figure 18.

What is a scalene triangle?

What is an isosceles triangle?
What is an equilateral triangle?

Sketch a scalene, isosceles, and equilateral triangle, as seen in Figure 19.

Work through Example 1 showing all work below.
Classify the given triangle as acute, obtuse, right, scalene, isosceles, or equilateral. State all that apply.

Watch the animation located on page 1.3-6 of the eText and explain why every isosceles right triangle has two acute angles that have a measure of $\frac{\pi}{4}$ radians.

NOW WORK SECTION 1.3 HW EXERCISES #1-2
Section 1.3 Objective 2 Using the Pythagorean Theorem

What is The Pythagorean Theorem? (Hint: See the text box on page 1.3-7.)

Work through Example 2 and show all work below.
Use the Pythagorean Theorem to find the length of the missing side of each of the given right triangles.

NOW WORK SECTION 1.3 HW EXERCISES #3-4
Section 1.3 Objective 3 Understanding Similar Triangles

What is the definition of similar triangles?

What are the Properties of Similar Triangles?

1.

2.

Work through the video accompanying Example 4 showing all work below.

Triangles ABC and XYZ are similar. Find the lengths of the missing sides of triangle ABC.
What is the definition of the **Proportionality Constant of Similar Triangles**?

Work through the animation accompanying Example 5 showing all work below.

The triangles below are similar. Find the proportionality constant. Then find the lengths of the missing sides.
Work through the video accompanying Example 6 showing all work below. The right triangles below are similar. Determine the lengths of the missing sides.

![Diagram of two similar right triangles]

NOW WORK SECTION 1.3 HW EXERCISES #5-8

Section 1.3 Objective 4 Understanding the Special Right Triangles

Watch the animation on page 1.3-18 which describes the \( \frac{\pi}{4}, \frac{\pi}{4}, \frac{\pi}{2} \) right triangle and take notes on how to establish a relationship between the lengths of the sides.
Sketch and label the $\frac{\pi}{4}, \frac{\pi}{4}, \frac{\pi}{2}$ right triangle as seen in Figure 24.

Watch the animation on page 1.3-19 which describes the $\frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}$ right triangle and take notes on how to establish a relationship between the lengths of the sides.
Sketch and label the $\frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}$ right triangle as seen in Figure 28.

Work through the interactive video with Example 7 and show all work below. Determine the lengths of the missing sides of each right triangle.

NOW WORK SECTION 1.3 HW EXERCISES #9-14
Section 1.3  Objective 5 Using Similar Triangles to Solve Applied Problems

Work through Example 8 and show all work below.

The shadow of a cell tower is 80 feet long. A boy 3 feet 9 inches tall is standing next to the tower. If the boy’s shadow is 6 feet long, find the height of the cell tower.

Work through the video with Example 9 and show all work below.

Two people are standing on opposite sides of a small river. One person is located at point Q, a distance of 20 feet from a bridge. The other person is standing on the southeast corner of the bridge at point P. The angle between the bridge and the line of sight from P to Q is $30^\circ$. Use this information to determine the length of the bridge and the distance between the two people. Round your answer to two decimal places as needed. (Note that you will need a calculator for this exercise. Only applications that do not require a calculator will be on your tests!)

NOW WORK SECTION 1.3 HW EXERCISES #15-16
Section 1.4

This Lesson will cover Section 1.4 in your e-Text. Work through each of the following tasks, carefully filling out the following pages.

Section 1.4 Right Triangle Trigonometry

☐ Work through TTK #1 then do problem #1
☐ Work through TTK #2 then do problem #2
☐ Work through TTK #3 then do problem #3
☐ Work through Objective 1 then do problems #4-9
☐ Work through Objective 2 then do problems #10-19
☐ Work through Objective 3 then do problems #20-25
Section 1.4 Right Triangle Trigonometry

1.4 Things To Know

1. Converting between Degree Measure and Radian Measure (Section 1.1)
Try working through a “You Try It” problem or refer to Section 1.1 or watch the animation.

NOW WORK SECTION 1.4 HW EXERCISE #1

2. Understanding Similar Triangles (Section 1.3)
Try working through a “You Try It” problem or refer to Section 1.3 or watch the video.

NOW WORK SECTION 1.4 HW EXERCISE #2

3. Understanding the Special Right Triangles (Section 1.3)
Try working through a “You Try It” problem or refer to Section 1.3 or watch the animation.

NOW WORK SECTION 1.4 HW EXERCISE #3
Section 1.4 Objective 1  Understanding the Right Triangle Definitions of the Trigonometric Functions

Watch the video seen at the top of page 1.4-3 and answer the questions below.

- Label the sides of this right triangle as seen in the video:

- Write down the Right Triangle Definitions of the Trigonometric Functions by filling in the blanks below:

Given a right triangle with acute angle $\theta$ and side lengths of hyp, opp, and adj, the six trigonometric functions of angle $\theta$ are defined as follows.

\[
\sin \theta = \phantom{0} \quad \csc \theta = \phantom{0}
\]

\[
\cos \theta = \phantom{0} \quad \sec \theta = \phantom{0}
\]

\[
\tan \theta = \phantom{0} \quad \cot \theta = \phantom{0}
\]

What silly phrase can help you to memorize the ratios for $\sin \theta, \cos \theta, \text{and} \tan \theta$?
Work through the interactive video with Example 1 showing all work below.
Given the right triangle evaluate the six trigonometric functions of the acute angle $\theta$.

**NOW WORK SECTION 1.4 HW EXERCISES #4-6**

Work through the video with Example 2 showing all work below.
If $\theta$ is an acute angle of a right triangle and if $\sin \theta = \frac{3}{4}$, then find the values of the remaining five trigonometric functions for angle $\theta$.

**NOW WORK SECTION 1.4 HW EXERCISES #7-9**
Section 1.4 Objective 2 Using the Special Right Triangles

Given the two special right triangles shown below, write down the side lengths as seen in Figure 32 on page 1.4-13. You may want to review these special right triangles by working through the animation as seen on page 1.4-13 of your eText.

Copy down the trigonometric functions for acute angles $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}$ as seen in Table 1.

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$\frac{\pi}{6}$ (30°)</th>
<th>$\frac{\pi}{4}$ (45°)</th>
<th>$\frac{\pi}{3}$ (60°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sin \theta$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\cos \theta$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tan \theta$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Work through the video with Example 3 and show all work below.

Determine the value of $\csc \frac{\pi}{6} + \cot \frac{\pi}{4}$.

NOW WORK SECTION 1.4 HW EXERCISES #10-13
Work through Example 4 and show all work below.
Determine the measure of the acute angle $\theta$ for which $\sec \theta = 2$.

NOW WORK SECTION 1.4 HW EXERCISES #14-19

Section 1.4 Objective 3 Understanding the Fundamental Trigonometric Identities

Watch the video shown on page 1.4-18, then write down The Quotient Identities.

Watch the video shown on page 1.4-19, then write down The Reciprocal Identities?
Work through Example 5 showing all work below.

Given that \( \sin \theta = \frac{5}{7} \) and \( \cos \theta = \frac{2\sqrt{6}}{7} \), find the values of the remaining four trigonometric functions using identities.

**NOW WORK SECTION 1.4 HW EXERCISES #20-21**

Watch the video shown on page 1.4-21, then write down The Pythagorean Identities?

Work through the interactive video with Example 6 showing all work below.

Use identities to find the exact value of each trigonometric expression.

a. \( \tan 37^\circ = \frac{\sin 37^\circ}{\cos 37^\circ} \)

b. \( \frac{1}{\cos^2 \frac{\pi}{9}} - \frac{1}{\cot^2 \frac{\pi}{9}} \)

**NOW WORK SECTION 1.4 HW EXERCISES #22-25**
Section 1.5

This Lesson will cover Section 1.5 in your e-Text. Work through each of the following tasks, carefully filling out the following pages.

Section 1.5 Trigonometric Functions of General Angles

☐ Work through Objective 1 then do problems #1-5
☐ Work through Objective 2 then do problems #6-9
☐ Work through Objective 3 then do problems #10-14
☐ Work through Objective 4 then do problems #15-22
☐ Work through Objective 5 then do problems #23-27
☐ Work through Objective 6 then do problems #28-40

Section 1.5 Trigonometric Functions of General Angles

Section 1.5 Objective 1 Understanding the Four Families of Special Angles

What is The Quadrantal Family of Angles? Watch the video on page 1.5-4 then sketch the angles shown in Figure 35.
What is the $\frac{\pi}{3}$ Family of Angles? Watch the video on page 1.5-5 then sketch the angles shown in Figure 36.

What is the $\frac{\pi}{6}$ Family of Angles? Watch the video on page 1.5-6 then sketch the angles shown in Figure 37.

What is the $\frac{\pi}{4}$ Family of Angles? Watch the video on page 1.5-7 then sketch the angles shown in Figure 38.

NOW WORK SECTION 1.5 HW EXERCISES #1-2
Work through the interactive video with Example 1 showing all work below.
Each of the given angles belongs to one of the four families of special angles. Determine the family of angles for which it belongs, sketch the angle, and then determine the angle of least nonnegative measure, $\theta_c$, coterminall with the given angle.

a. $\theta = \frac{29\pi}{6}$

b. $\theta = \frac{14\pi}{2}$

c. $\theta = -\frac{18\pi}{4}$

d. $\theta = \frac{11\pi}{4}$

e. $\theta = \frac{14\pi}{6}$

f. $\theta = 420^\circ$

g. $\theta = -495^\circ$

NOW WORK SECTION 1.5 HW EXERCISES #3-5
Section 1.5 Objective 2 Understanding the Definitions of the Trigonometric Functions of General Angles

Work through the animation seen on page 1.5-12 and take notes here.

What are The General Angle Definitions of the Trigonometric Functions?

Under what conditions will the following trigonometric functions be undefined (if ever)?

\[ \tan \theta = \frac{y}{x} \quad \text{and} \quad \sec \theta = \frac{r}{x} : \]

\[ \csc \theta = \frac{r}{y} \quad \text{and} \quad \cot \theta = \frac{x}{y} : \]

\[ \sin \theta = \frac{y}{r} \quad \text{and} \quad \cos \theta = \frac{x}{r} : \]

Work through the video with Example 2 and show all work below.
Suppose that the point \((-4, -6)\) is on the terminal side of an angle \(\theta\). Find the six trigonometric functions of \(\theta\).

NOW WORK SECTION 1.5 HW EXERCISES #6-9
Section 1.5 Objective 3 Finding the Values of the Trigonometric Functions of Quadrantal Angles

Work through the video as seen on page 1.5-17 and take notes here. Then fill out Table 2 below:

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$\sin \theta$</th>
<th>$\cos \theta$</th>
<th>$\tan \theta$</th>
<th>$\csc \theta$</th>
<th>$\sec \theta$</th>
<th>$\cot \theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{\pi}{2}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\pi$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{3\pi}{2}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Work through the interactive video accompanying Example 3 showing all work below.
Without using a calculator, determine the value of the trigonometric function or state that the value is undefined.

a. \( \cos(-11\pi) \)

b. \( \csc(-270^\circ) \)

c. \( \tan\left(\frac{13\pi}{2}\right) \)

d. \( \sin(540^\circ) \)

e. \( \cot\left(\frac{7\pi}{2}\right) \)

NOW WORK SECTION 1.5 IIW EXERCISES #10-14
Section 1.5 Objective 4 Understanding the Signs of the Trigonometric Functions

Watch the video as seen on page 1.5-21 and answer the questions below:

The sign of each trigonometric function is determined by the ____________ in which the terminal side of the angle lies.

Which trigonometric functions are positive for all angles with a terminal side lying in the following quadrants?

Quadrant I:

Quadrant II:

Quadrant III:

Quadrant IV:

What acronym can help us remember the signs of the trigonometric functions for angles whose terminal side lies in one of the four quadrants?

Sketch the diagram shown in Figure 49.
Three grids are shown below. (See Figure 50.) The first grid represents the sign of the values of $y = \sin x$ in each quadrant. The middle grid represents the sign of the values of $y = \cos x$. The third quadrant represents the sign of the values of $y = \tan x$ in each quadrant. Place a “+” or “−” in each quadrant of each grid to represent the appropriate sign.

NOW WORK SECTION 1.5 HW EXERCISES #15-18
Work through the video with Example 4 and show all work below.
Suppose $\theta$ is a positive angle in standard position such that $\sin \theta < 0$ and $\sec \theta > 0$.

a. Determine the quadrant in which the terminal side of angle $\theta$ lies.

b. Find the value of $\tan \theta$ if $\sec \theta = \sqrt{5}$.

NOW WORK SECTION 1.5 HW EXERCISES #19-22
Section 1.5 Objective 5 Determining Reference Angles

What is the definition of the Reference Angle?

The measure of the \( \theta_b \) depends on the quadrant in which the \( \theta_c \) lies.
The four cases for reference angles are shown below. Fill in the blanks. (See page 1.5-31 of your eText.)

**Case 1:** If the terminal side of \( \theta_C \) lies in Quadrant I, then \( \theta_R = \) _____.

**Case 2:** If the terminal side of \( \theta_C \) lies in Quadrant II, then \( \theta_R = \) (or \( \theta_R = \) _____).

**Case 3:** If the terminal side of \( \theta_C \) lies in Quadrant III, then \( \theta_R = \) _____ (or \( \theta_R = \) _____).

**Case 4:** If the terminal side of \( \theta_C \) lies in Quadrant IV, then \( \theta_R = \) _____ (or \( \theta_R = \) _____).
Work through the interactive video with Example 5 and show all work below.
For each of the given angles, determine the reference angle.

a. \( \theta = \frac{5\pi}{3} \)

b. \( \theta = \frac{11\pi}{4} \)

c. \( \theta = -\frac{25\pi}{6} \)

d. \( \theta = \frac{16\pi}{6} \)

NOW WORK SECTION 1.5 HW EXERCISES #23-25
Work through the interactive video with Example 6 and show all work below.
For each of the given angles, determine the reference angle.

a. \[ \theta = \frac{5\pi}{8} \]

b. \[ \theta = \frac{22\pi}{9} \]

c. \[ \theta = -\frac{5\pi}{7} \]

NOW WORK SECTION 1.5 HW EXERCISES #26-27

Section 1.5. Objective 6 Evaluating Trigonometric Functions of Angles Belonging to the \( \pi/3 \), \( \pi/6 \), or \( \pi/4 \) Families.
Work through the interactive video with Example 8 and show all work below.

Find the values of the six trigonometric functions for $\theta = \frac{7\pi}{4}$.

What are the four Steps for Evaluating Trigonometric Functions of Angles Belonging to the $\frac{\pi}{3}$, $\frac{\pi}{6}$, or $\frac{\pi}{4}$ Families?

Step 1:

Step 2:

Step 3:

Step 4:

Work through the interactive video with Example 9 and show all work below.

Find the exact value of each trigonometric expression without using a calculator.
a. $\sin \left( \frac{7\pi}{6} \right)$

b. $\cot \left( \frac{22\pi}{3} \right)$

c. $\tan \left( \frac{11\pi}{4} \right)$

d. $\cos \left( \frac{11\pi}{3} \right)$
e. \( \sec \left( \frac{5\pi}{6} \right) \)

f. \( \csc \left( -\frac{7\pi}{6} \right) \)

NOW WORK SECTION 1.5 HW EXERCISES #28-40