Maximum stand-density model development and validation

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Site Type Initiative

- Phase I: Data assembly, model development, validation
- Phase II: Field studies
Goal of forest stand density management studies

Accurately predict site-specific maximum stand density across the Inland Northwest to meet landowner management objectives

- Forest size-density: conventions, assumptions
- Modeling approach
- Factors controlling size-density functions
- Validating the unmeasurable
- Using the predictions
The size-density function

- Non-linear exponential function
- Log transformed into linear function

\[ y = \beta_0 e^{\beta_1 x} \]

\[ \ln(\text{TPH}) = \beta_0 + \beta_1 \ln(\text{QMD}) \]
Upper boundary defines maximum stand density

*The self-thinning line*

- Two conventions for tracking stand development
- Diameter increases until stand nears maximum stand density
  i.e. imminent mortality growth phase

I. Density Management Diagrams

II. Stand density index
SDI_{max} = e^{(\beta_0 + \beta_1 \cdot \ln 10)}

Assumes:
• Slope is universal (-1.605)
• Intercept is constant for a given species and region
  — i.e. not affected by site factors
Are the assumption valid?

- Slope is not universally -1.605
- Site does affect the intercept

Weiskittel et al 2009

Powell 1999
Objective of modeling approach:

Identify and employ the most effective approach to define the principal factors that control the size-density function
Data assembly

Dataset:
>110,000 plots
4+ million trees
28 tree species

Associated Input:
Sand/tree level, climate, geology, topography

Cooperator Data Suppliers:
Bennett Lumber, BLM, Forest Capital, Hancock, IDL, Inland Empire Paper, Stimson, USFS-FIA/CVS, WA DNR
Fitting the size-density function limit

*The self-thinning line*

Ordinary least squares regression (OLS)

Fit a line through all points, then shift to the top edge

Select stands with highest SDI, then fit a line through those points
Fitting the size-density line with Stochastic Frontier Regression (SFR)

The SFR Model:

\[ \ln(\text{TPA}) = \beta_0 + \beta_1 \ln(\text{QMD}) + \nu - u \]

\( \nu \) = two-sided random error

\( u \) = non-negative random error

Maximum likelihood techniques are used to estimate the frontier.

- Combined error defines the frontier
- Multi-variable analysis possible
- Describes variability above the size-density line
SFR detects species differences in max SDI

Basic model applied to each of four species:

\[ \ln(TPA) = b_0 + b_1 \cdot \ln(QMD) + v + u \]

<table>
<thead>
<tr>
<th>Species</th>
<th>Intercept</th>
<th>Slope</th>
<th>SDI MAX</th>
</tr>
</thead>
<tbody>
<tr>
<td>Douglas-fir</td>
<td>9.878 (0.0106)</td>
<td>-1.515 (0.0239)</td>
<td>596</td>
</tr>
<tr>
<td>Grand-fir</td>
<td>9.852 (0.0195)</td>
<td>-1.511 (0.0094)</td>
<td>585</td>
</tr>
<tr>
<td>Ponderosa Pine</td>
<td>10.256 (0.03022)</td>
<td>-1.777 (0.0123)</td>
<td>475</td>
</tr>
<tr>
<td>Western larch</td>
<td>9.24421 (0.03550)</td>
<td>-1.237 (0.01761)</td>
<td>599</td>
</tr>
</tbody>
</table>

Select when target species is more than 20% of plot basal area
So, includes abundant mixed species stands
Testing site factors:

Are the self-thinning lines affected by soil parent material?

Rock-type covariates added to individual species models:

\[
Ln(TPA) = b_0 + b_1 \cdot Ln(QMD) + b_{2i} \cdot \text{Rock Type}_i + \nu + \mu
\]

<table>
<thead>
<tr>
<th>Rock Type</th>
<th>Intercept</th>
<th>Slope</th>
<th>SDI Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>CaMetased</td>
<td>9.96</td>
<td>-1.55</td>
<td>596</td>
</tr>
<tr>
<td>Extrusive</td>
<td>10.22</td>
<td>-1.66</td>
<td>606</td>
</tr>
<tr>
<td>Glacial</td>
<td>9.84</td>
<td>-1.53</td>
<td>557</td>
</tr>
<tr>
<td>Intrusive</td>
<td>9.85</td>
<td>-1.53</td>
<td>568</td>
</tr>
<tr>
<td>Metasedimentary</td>
<td>9.78</td>
<td>-1.48</td>
<td>588</td>
</tr>
<tr>
<td>Sedimentary</td>
<td>9.88</td>
<td>-1.52</td>
<td>585</td>
</tr>
</tbody>
</table>

Soil parent material affects both slope and intercept of self-thinning line
Testing site factors:
*Are the self-thinning lines affected by physiography and climate?*

Physiography:
- Elevation
- Slope
- Aspect

Climate:
- Clusters of variables represented by:
  - Annual degree-days >5 °C: \(dd5\)
  - Frost-free period: \(ffp\)
  - Mean temp coldest month: \(mtcm\)
  - Annual Dryness Index: \(adi\)
  - (temp/precip)
  - Summer/Spring precip balance: \(sspb\)
Elevation

Mean temperature of coldest month, mtcm
Percent basal area influences individual species models

- Implies that occurrence of other species increases maximum density over that in pure stands
- Must account for the influence of other species when present
Mixed species model

- Full model includes rocks, physiography, climate and species

\[
Ln(TPA) = b_0 + b_1 \cdot Ln(QMD) + b_{2i} \cdot Rock \ Type_i + b_{3j} \cdot Physiography_j + b_{4k} \cdot Climate_k + b_{5m} \cdot Species \ BA_m + v + u
\]

- 10% of data reserved for validation
Individual species estimates obtained by specifying pure stands

- Greater stocking is possible in mixed species stands
Validating model estimates

- Measuring maximum density is difficult
- Must be observed in stands that are naturally self-thinning
- Repeated measurements are most helpful
- Even then, differentiating stands may not be at 100% stocking
Validation

Compare 10% reserved data to model estimates

- Mean of upper quartile is 100% SDI max
- Or, 12.5% of plots have stocking levels above SDI max
- Consequence of SFR error terms

\[ u \text{ is one sided error} \]
\[ v \text{ is two sided error as in OLS} \]
Validation

Compare 10% reserved data to model estimates

Contrasting MTCM

Pure DF

Select stands with highest or lowest MTCM

Pure DF vs. Mixed Species

Select pure Douglas-fir for comparison with mixed stands
Validation with long-term measurement plots

*Forest Service 100-yr data*
Testing the assumptions

*What precision is required?*

- Species predominately controls maximum density models
- Site has some influence
- The slope is not universally -1.605
Implications from maximum density predictions

- Known factors control stocking, and therefore productivity rates and yields
- Species mix alters stocking more than site factors
- Accounting for other species becomes important
  - Especially when it’s hard to maintain pure stands
- Density management diagrams
  - Slope adjustments necessary
  - Intercept differences among sites are more subtle
- Temperature and elevation do define stocking; therefore:
  - Climate change will affect stocking
  - IFTNC maximum density models predict climate change effect for a give species mix
Next steps

• Model and data adjustments to improve validation
  – Adjust SFR parameters to push estimates closer to frontier
  – Screen dataset for outlier plots

• Potential long-term datasets are important for validation
  – BC Ministry
  – Updated IDL Continuous Inventory Plots
  – Others?