Independent Study in Idaho

MATH 160
Survey of Calculus

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The University of Idaho in statewide cooperation with Boise State University — Idaho State University — Lewis-Clark State College
Course Guide

Independent Study in Idaho

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Mathematics 160
Survey of Calculus

University of Idaho
4 Semester-Hour Credits

Prepared by:
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Associate Professor of Mathematics
Southern Oregon University

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2 -Math 160
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Math 160: Survey of Calculus 4 Semester-Hour Credits: UI

Welcome!
Whether you are a new or returning student, welcome to the Independent Study in Idaho (ISI) program. Below, you will find information pertinent to your course including the course description, course materials, course objectives, as well as information about lessons, exams, and grading.

Policies and Procedures
Refer to the ISI website at www.uidaho.edu/isi and select Students for the most current policies and procedures, including information on setting up accounts, student confidentiality, exams, proctors, transcripts, course exchanges, refunds, academic integrity, library resources, and disability support and other services.

Course Description
Overview of functions, and graphs, derivatives, integrals, exponential and logarithmic functions, functions of several variables and differential equations. Primarily for students who need only one semester of calculus, such as students in business, or architecture. Prerequisite: sufficient score on SAT, ACT, or math placement test, or Math 137 Algebra with Applications with a C or better, or Math 143 with a C or better. Required test scores can be found here: www.uidaho.edu/registrar/registration/placement/. UI students: Polya Math Center unavailable for ISI students; carries no credit after Math 170; General education: Mathematics.

Recommended: non-graphing calculator
20 graded assignments, 8 self-study assignments, 5 proctored exams
Be advised, exams for this course are sent one-at-a-time once appropriate lessons have been graded.

Students may submit up to 2 assignments per week. Before taking exams, students MUST wait for grades and feedback on assignments, which may take up to three weeks after date of receipt by the instructor.

ALL assignments and exams must be submitted to receive a final grade for the course.

Course Materials
Required Course Materials

Supplemental Materials (optional, but recommended):

- Texas Instrument TI-30X IIS 2-Line Scientific Calculator ONLY. No other calculators allowed for test-taking (see image to right).
Course Delivery
All ISI courses are delivered through BbLearn, an online management system that hosts the course lessons and assignments and other items that are essential to the course. Upon registration, the student will receive a Registration Confirmation Email with information on how to access ISI courses online.

Course Introduction
It is assumed that the student taking this course has already mastered the essential elements of algebra and Euclidean geometry. An excellent algebra review is given in Chapter 0 of the text, but basic geometric concepts such as point, line, plane, and area formulas for plane figures such as circles, rectangles, and triangles are taken for granted. Trigonometric concepts such as sine and cosine are not used at all in this course. A prior understanding of exponential and logarithmic functions is desirable, although these topics are introduced under the assumption that the student has had little or no prior exposure to them.

The calculus introduced here is at an intuitive level. Essential definitions and postulates are given as in any beginning calculus course, but theorems are stated without formal proof. This course stresses application. Since the ability to immediately apply calculus, especially in the areas of business and economics, is the primary objective of the course, problem-solving will be stressed in both the homework and on the examinations.

Course Objectives
- The objective of this course is to learn about integral calculus and its applications, with a concentration on business applications.
- An understanding of the applications of integral calculus will strengthen the student’s ability to tackle problems analytically.
- Although the following is not qualitative, it illustrates approximately what type of work is expected for the letter grades A, B, and C. Students earning an “A” will have completed all lessons (this includes self-study lessons) in an exceptional fashion. In addition they may have exhibited few algebraic errors in their work, used proper notation throughout the course, properly labeled all graphs, and mastered each of the topics covered by the lessons. Students earning a “B” will have successfully completed all lessons. They may have exhibited some algebraic errors in their work, with improvements made along the way in algebraic skills, notation, and graphs, and have mastered almost all of the topics covered by the lessons. Students earning a “C” will have satisfactorily worked on all the lessons and shown the ability to improve their algebraic skills and demonstrated extensive knowledge of each of the topics covered by the lessons.

Lessons
Each lesson includes the following components:
- A reading assignment
- An introduction
- Comments
- A written assignment
- A checklist of topics

You will be required to show all work necessary to solve each problem or you will receive no credit.

Some lessons in this course are self-study. The exercises in these lessons are for you to see how well you have mastered the material in the text and this study guide. Read the assignments carefully before beginning the self-study questions; answer them to the best of your ability, and then check your answers in the supplement to the textbook. Although these lessons will not be submitted for grading, the material may be covered in the examinations. Do your best and do not skip these important lessons.
Use of the Study Guide
Each lesson begins with a reading assignment followed by a brief introduction. Sections of the text not specifically mentioned should be omitted. It is important that you thoroughly understand the material presented in these reading assignments, particularly the examples, prior to proceeding with the lesson. Following the introduction is a section titled Comments. The comments section of a lesson may vary in length from a few sentences to several pages with worked examples.

Practice assignments are given next. These consist primarily of the odd-numbered exercises. There are some detailed solutions to practice exercises in the text’s supplement, *Study Guide & Selected Solutions Manual, Brief Calculus & Its Applications*. In addition, you will find a few similar problems worked out in detail in this study guide.

The written assignment is the last part of each lesson. This consists primarily of the even-numbered exercises. Please note that it is your responsibility to provide the work and detail necessary to obtain and justify your solutions.

Study Hints:
• Keep a copy of every lesson submitted.
• Complete all lessons including the reading and self-study lessons.
• Set a reasonable schedule allowing for completion of the course one month prior to your desired deadline.
• Use proper notation and include scales on all graphs in the lessons.
• Expect this course to take approximately four months, but possibly longer, assuming you work on it every day.
• Set a schedule allowing for completion of the course one month prior to your desired deadline. (An Assignment Submission Log is provided for this purpose.)

Exams
• You must wait for grades and comments on lessons prior to taking each subsequent exam.
• For your instructor’s exam guidelines, refer to the Exam Information sections in this study guide.

At the appropriate times, you will be directed to take an examination before proceeding with additional lessons. All examinations must be taken without the use of the text, this study guide, or notes of any sort (you may use a calculator, but not a graphing calculator). Therefore, the concepts, definitions, theorems, etc., required to do the assignments must be committed to memory. You will not be asked to state a definition verbatim or to prove a theorem, but you will be expected to use them in solving both theoretical and application-type problems. Don’t panic! You will find the rules of calculus are very few and quite easy to use. The typical student has much more difficulty remembering the fundamentals of algebra than he or she does remembering the fundamentals of calculus learned here.

Each of the four 2-hour examinations will cover only that material since the last examination. For the most part, 80 to 90 percent of each examination will consist of questions similar to those in the assigned problems. The remaining test questions will see if you can apply your newfound knowledge to new situations.

The final examination will be comprehensive, and you will be given three hours to complete it. The problems will be similar in structure and type to the two-hour examinations.

It should be stressed that your success on the two-hour examinations, and on the final, will depend on how diligently you work through all the lessons, including the practice problems. **It is recommended that you wait until the lessons are returned before taking the examination over a particular section.** This way you are sure that procedures used to solve the problems are correct. You have only one chance at each
exam, so be sure that you are prepared and have all the lessons completed and returned to you.

See *Grading* for specific information on exams, points, and percentages.

**Proctor Selection/Scheduling Exams**

All exams require a proctor. At least 2 weeks prior to taking your first exam, submit the completed *Proctor/Exam Request Form* (available at uidaho.edu/isi, under *Forms*) to the ISI office. ISI mails all exams directly to the proctor after receiving the *Proctor/Exam Request Form*. You must schedule the examination time with your proctor prior to each exam. The proctor administers the exam and returns it to the ISI office.

**Grading**

The course grade will be based upon the following considerations:
The grading for this course will be based on 800 points, broken down as follows:

<table>
<thead>
<tr>
<th>Assignments (20)</th>
<th>200 points: (10 points each)</th>
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<tbody>
<tr>
<td>Examinations (4)</td>
<td>400 points (100 points each)</td>
</tr>
<tr>
<td>Final Examination (Comprehensive)</td>
<td>200 points</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>800 points possible</td>
</tr>
</tbody>
</table>

Grades will be assigned according to the following:

<table>
<thead>
<tr>
<th>Grade</th>
<th>Points Range</th>
<th>Percentage</th>
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<tbody>
<tr>
<td>A</td>
<td>720 to 800</td>
<td>90%+</td>
</tr>
<tr>
<td>B</td>
<td>640 to 719</td>
<td>80%+</td>
</tr>
<tr>
<td>C</td>
<td>560 to 639</td>
<td>70%+</td>
</tr>
<tr>
<td>D</td>
<td>480 to 559</td>
<td>60%+</td>
</tr>
<tr>
<td>F</td>
<td>479 or fewer</td>
<td>below 60%</td>
</tr>
</tbody>
</table>

The final course grade is issued after all lessons and exams have been graded.

Acts of academic dishonesty, including cheating or plagiarism are considered a very serious transgression and may result in a grade of F for the course.

**About the Course Developer**

This course was developed by Dr. Dusty E. Sabo, an Associate Professor of Mathematics at Southern Oregon University in Ashland, Oregon. In 1996 he obtained his Ph.D. from the University of Idaho where he studied combinatorial geometry under the direction of Professor Mark J. Nielsen. He has worked as an instructor for Independent Study in Idaho since 1994.

**Contacting Your Instructor**

Instructor contact information is posted in the *Course Rules* document on your BbLearn site.
### Assignment Submission Log

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<tr>
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<tr>
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<td>0</td>
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<td>assigned problems</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>Sections 1.1, 1.2</td>
<td>assigned problems</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>Sections 1.3, 1.4</td>
<td>assigned problems</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>Sections 1.5, 1.6</td>
<td>assigned problems</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>Sections 1.7, 1.8</td>
<td>assigned problems</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0, 1</td>
<td>Review</td>
<td>assigned problems</td>
<td>self-study</td>
</tr>
</tbody>
</table>

**It is time to make arrangements with your proctor to take Exam 1.**

| 8      | 2       | Sections 2.1, 2.2  | assigned problems  | self-study    |
| 9      | 2       | Sections 2.3, 2.4  | assigned problems  |               |
| 10     | 2       | Section 2.5        | assigned problems  |               |
| 11     | 2       | Section 2.6        | assigned problems  |               |
| 12     | 2       | Section 2.7        | assigned problems  |               |
| 13     | 3       | Sections 3.1, 3.2  | assigned problems  |               |
| 14     | 2, 3    | Review             | assigned problems  | self-study    |

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<tr>
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<td>4</td>
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<td>4</td>
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<td>5</td>
<td>Section 5.2</td>
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<td>6</td>
<td>Section 6.6</td>
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<td>26</td>
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<td>self-study</td>
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It is time to make arrangements with your proctor to take Exam 4.

It is time to make arrangements with your proctor to take the Final Exam.
Lesson 1
Self-Study: Preliminary Concepts

Do not submit this lesson for grading. After carefully working through the following questions, check your answers with the solutions found in the textbook.

Reading Assignments
Chapter 0, Section 0.1, pp. 3-15; Section 0.2, pp. 18-25.

Introduction
This course begins with a review of some of the fundamental concepts needed for the study of calculus. This first lesson explores the concept of a function, functional notation, and the graph of a function.

Comments
These sections briefly review the concept of a function. The importance of being able to use and understand function notation throughout this course cannot be overstated. In particular, note at the bottom of page 7 the use of the convention in higher mathematics that if the domain of a function is not explicitly stated, it is implied to “consist of all numbers for which the formula defining the function makes sense.” For this class’s needs, domains of the functions will always be restricted to subsets of the real numbers. For example, making sense means that mathematically excluded from our domains are numbers yielding division by zero, or numbers requiring the square root of negative numbers. Make these exclusions, as only on occasion will they be explicitly stated. Making sense means even more with regard to word problems where mathematical modes are created for real-world situations. Restrict domains not only to numbers that make mathematical sense, but also to those numbers that are reasonable in the context of the problem.

Frequently, numbers that make mathematical sense do not make sense as the solutions to problems which represent real-life situations, such as determining optimal production levels in a factory. Mathematically, a formula may result in the numbers -150 and 200. Eliminate -150 since, in most cases, production levels are not thought of in negative terms.

In Section 0.2, pay particular attention to the discussion of linear functions. The ability to understand the algebra of straight lines is important to understanding the notion of a derivative found in calculus.

Self-Study Assignments
Section 0.1: 13, 15, 17, 19, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 47, 53 (pp. 15-17).
Section 0.2: 1, 3, 7, 9, 17, 21, 29 (pp. 26-27).

Checklist Lesson 1
Chapter 0, Section 0.1

Function
Value of a function
Domain of a function
Graph of a function
Vertical line test
Chapter 0. Section 0.2

Linear function
Constant function
$x$- and $y$-intercepts
Quadratic function
Polynomial function
Power function
Absolute value function
Lesson 2
Preliminary Concepts, Continued

Reading Assignments
Chapter 0, Section 0.3 (pp. 27-30); Chapter 0, Section 0.4 (pp. 32-39); Chapter 0, Section 0.5 (pp. 40-46).

Introduction
In this lesson, concepts involving functions are further developed and basic properties of exponents are reviewed.

Comments
Frequently encountered are Functions that can be viewed as combinations of other functions are frequently encountered. Section 0.3 reviews the algebraic techniques needed to combine functions by addition, subtraction, multiplication, and division.

Study the first four examples carefully. These examples will also help in reviewing some basic algebra. Another important method of combining two functions \( f(x) \) and \( g(x) \) is to substitute the function \( g(x) \) for every occurrence of the variable \( x \) in \( f(x) \). Think of the composition of a function as the \textit{function of a function}. For example, if

\[
f(x) = x^2 + 2x + 1 \quad \text{and} \quad g(x) = x + 3 \quad \text{then}
\]

\[
f(g(x)) = f(x + 3) = (x + 3)^2 + 2(x + 3) + 1.
\]

Note the difference between \( f(g(x)) \), the composition of two functions, and \( f(x)g(x) \), the product of two functions; \textit{in general, they are not the same}.

Section 0.4 reviews factoring and the quadratic formula. Again, the problems in this section will help in reviewing basic algebra. \textbf{The quadratic formula should be memorized.} Always try factoring as a primary method of solving quadratic equations. If factoring does not work, use the quadratic formula. This course is restricted to the real number system, so imaginary numbers as possible solutions to quadratic equations are not used here. \textbf{A zero of a function} \( f(x) \) \textbf{is a value for} \( x \) \textbf{for which} \( f(x) = 0 \).

Section 0.5 reviews the operations with exponents that are used throughout the text. It is necessary to be thoroughly familiar with the laws of exponents as seen on page 41. If you have never worked with rational exponents before, do not panic. At this point in your mathematical career, you have covered all the concepts where rational exponents prove useful, but have probably used other equivalent notation. The use of rational exponents is not difficult to master, but it is critical to the work we will be doing from now on.

Practice Problems

Section 0.3: 1, 3, 5, 7, 13, 15, 25, 27, 31, 33 (pp. 31-32).
Section 0.4: 1, 7, 13, 19, 25, 27, 29, 33 (pp. 39-40).
Section 0.5: 1, 3, 7, 9, 17, 19, 21, 25, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61, 63, 65, 69 (pp. 46-47).

Written Assignment

Section 0.3: 2, 4, 6, 8, 14, 16, 26, 30, 34 (pp. 31-32).
Section 0.4: 2, 8, 14, 16, 18, 20, 26, 28, 30, 34 (pp. 39-40).
Section 0.5: 2, 4, 10, 14, 16, 18, 26, 42, 44, 46, 48, 50, 52, 54, 56, 58, 60, 62, 64, 66, 68, 70 (pp. 46-47).
Checklist Lesson 2
Chapter 0, Section 0.3

Addition, subtraction, multiplication, and division of functions
Composition of functions

Chapter 0, Section 0.4

Zero of a function
Quadratic formula

Chapter 0, Section 0.5

Factoring polynomials
Laws of exponents
Lesson 3
Slopes of Straight and Curved Lines

Reading Assignments
Chapter 1, Section 1.1, pp. 68-75; Section 1.2, pp. 79-81.

Introduction
Calculus can be broken down into two branches: differential calculus and integral calculus. At the heart of differential calculus, which is studied in the next five chapters of this text, lies the concept of the derivative.

In the first two sections of this chapter, the concept of the slope of a straight line is reviewed and this concept of the slope is extended to more general curves.

Comments
The first section of this lesson reviews the properties of the slope of a straight line. The study of straight lines is crucial for the study of the steepness of curves. Be thoroughly familiar with all properties discussed. For emphasis, some of the properties are restated below.

Slope Property 2
If \((x_1, y_1)\) and \((x_2, y_2)\) are on the line, then the slope of the line is

\[
m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{(slope formula)}.
\]

As we move from \((x_1, y_1)\) to \((x_2, y_2)\), the change in y-coordinates is \(y_2 - y_1\) and the change in x-coordinates is \(x_2 - x_1\). The slope is simply the ratio of the change in y to the change in x; in other words the slope of the line equals the change in y per unit change in x, so we say the slope gives the rate of change of y with respect to x.

Slope Property 3
If the slope of the line is \(m\) and if \((x_1, y_1)\) is a point on the line, then the equation of the line is

\[
y - y_1 = m (x - x_1)
\]

This equation is called the point-slope form of the equation of the line.

In the second section of this lesson, the concept of the slope is extended to more general curves. Carefully read the discussion at the beginning of this section.

The slope of a curve at a point \(P\) is defined as the slope of the tangent line to curve at the point \(P\). The two expressions, “the slope of a curve at a point \(P\)” and “the slope of a tangent line to the curve at point \(P\)” mean the same thing and are used interchangeably.

Practice Problems
Section 1.1: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 29, 33, 35, 43, 45, 53 (pp. 76-78)
Section 1.2: 1, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27 (pp. 82-85).

**Written Assignment**
Section 1.1: 2, 4, 6, 8, 10, 12, 14, 16, 18, 22, 24, 34, 36, 44, 48, 49, 54, 55 (pp. 76-78).
Section 1.2: 2, 4, 8, 12, 14, 16, 18, 20, 22, 24, 26, 28 (pp. 82-85).

**Checklist Lesson 3**
Chapter 1, Section 1.1
- Slope of a line
- y-intercept
- Slope properties 1 to 5

Chapter 1, Section 1.2
- Slope of a curve at a point
Lesson 4
Derivatives and Limits

Reading Assignments
Chapter 1, Section 1.3, pp. 85-91; Section 1.4, pp. 94-101.

Introduction
In the first section of the lesson, the concept of the derivative is discussed, the derivatives of several simple functions are given, and the initial concepts needed in the definition of the derivative are presented.

In the second section of this lesson, the derivative is defined using limits, one of the most important concepts in all of calculus.

Comments
In Section 1.3, the concept of the derivative is discussed intuitively. Suppose a curve is the graph of a function \( f(x) \). It is usually possible to obtain a formula that gives the slope of the curve \( y = f(x) \) at any point. This slope formula is called the derivative of \( f(x) \) and is written \( f'(x) \). The derivative formulas presented here are either presented intuitively or are stated without proof. The rules are summarized below:

- If \( f(x) = mx + b \) then \( f'(x) = m \)
- If \( f(x) = b \) then \( f'(x) = 0 \)
- If \( f(x) = x^2 \) then \( f'(x) = 2x \)
- If \( f(x) = x^3 \) then \( f'(x) = 3x^2 \)

Power Rule

- If \( f(x) = x^r \) then \( f'(x) = rx^{r-1} \) (Where \( r \) is any real number)
- If \( f(x) = \frac{1}{x} \) then \( f'(x) = -\frac{1}{x^2} \) (This last formula is derived by using the power rule where \( r = -1 \.)

Study example 2, on page 88 carefully. Be careful. Read the warning and be sure not to confuse \( f(x) \) and \( f'(x) \). Many beginning students of calculus write \( f(x) = f'(x) \). This statement is almost always false!

Notation

The symbol \( \frac{d}{dx} \) (the derivative with respect to \( x \)) indicates the operation finding the derivative of \( f(x) \),

\[
\frac{d}{dx} \left[ f(x) \right] = f'(x).
\]

Never write \( \frac{d}{dx} = \ldots \). Instead use \( \frac{dy}{dx} = \ldots \) or \( f'(x) = \ldots \).

Also, when working with an equation of the form \( y = f(x) \), we often write \( \frac{dy}{dx} \) as a symbol for the derivative \( f'(x) \). Become familiar with these different notation conventions.
The Secant Line Calculation of the Derivative

Follow the discussion on pages 88-90 very carefully. The steps on page 90 are restated here and more examples are worked.

To calculate \( f'(x) \):
1. First calculate \( \frac{f(x + h) - f(x)}{h} \), \( h \neq 0 \)
2. Take the limit as \( h \) approaches zero (\( h \to 0 \)).
3. The quantity \( \frac{f(x + h) - f(x)}{h} \) will approach \( f'(x) \) as \( h \) goes to zero.

Example Verification of Rule 1: If \( f(x) = mx + b \), \( f'(x) = m \).
First this will be done for a specific function. Then the formula will be derived in general.

Let \( f(x) = 3x + 2 \) and show that \( f'(x) = 3 \).

\[
\frac{f(x + h) - f(x)}{h} = \frac{3(x + h) + 2 - (3x + 2)}{h}
\]

As \( h \) approaches zero, the quantity approaches 3 (since this quantity doesn’t contain an \( h \), it does not change as \( h \) approaches zero); thus we have \( f'(x) = 3 \).

Let \( f(x) = mx + b \), \( f(x + h) = m(x + h) + b \)

\[
\frac{f(x + h) - f(x)}{h} = \frac{m(x + h) + b - (mx + b)}{h}
\]

As \( h \) approaches zero, the quantity approaches \( m \); therefore, \( f'(x) = m \), the slope of \( f(x) \).

In Section 1.4, the derivative is defined using limits.

\[
f'(a) = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}
\]

In order to better understand the limit concept used in defining the derivative, limits are discussed in a more general setting. It is not unusual for students to feel uncomfortable when dealing with limits.
Let \( g(x) \) be a function, “\( a \)” a number. We say that the number \( L \) is the limit of \( g(x) \) as \( x \) approaches \( a \) provided that as \( x \) gets arbitrarily close to \( a \) (but not equal to \( a \)) the values of \( g(x) \) approach \( L \). \( (L \) is a finite number; also, the value of \( g(x) \) may actually equal \( L \).) In this case, we write \( \lim_{x \to a} g(x) = L \). If, as \( x \) approaches \( a \), the value of \( g(x) \) does not approach a specific finite number, then we say that the limit of \( g(x) \) as \( x \) approaches \( a \) does not exist.

Read the samples carefully. (We will not do Infinity and Limits on page 100.)

Study example 2, on page 96 carefully.

At first it is not unusual for students to have difficulty determining a limit by looking at the graph of a function. Suppose we have a function \( g(x) \). The graph of the function is obtained by graphing the set of all points \([x, g(x)]\). So the values of the function for certain values of \( x \) are obtained from the graph by looking at the \( y \)-coordinates of the points.

Also, the open circles drawn on the graphs are meant to represent breaks in the graphs, indicating that the functions under consideration are not defined at the corresponding value of \( x \). A solid circle indicates that the function is defined for the corresponding value of \( x \), and the value of the function is the corresponding \( y \)-coordinate of the point.

In example 2(c) the graph appears as follows:

![Graph of a function](image)

The vertical dashed line is called a vertical asymptote. As values of \( x \) approach 2 from either side, the value of the function \( g(x) \) increases without bound (in this case the value of the function approaches \( +\infty \)).

Reread the definition of a limit carefully. The \( \lim_{x \to a} g(x) \) does not depend on \( g(a) \).
Examine the following carefully.

\[ h(x) = \begin{cases} \frac{x^2 - 1}{x - 1} & \text{if } x \neq 1 \\ 4 & \text{if } x = 1 \end{cases} \]

As \( x \) approaches 1 (look at a sequence of values for \( x \) that get closer and closer to 1 but are not equal to 1), the values of the function get closer and closer to 2. Therefore, \( \lim_{{x \to 1}} h(x) = 2 \), while \( h(2) = 4 \).

**Practice Problems**
Section 1.3: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 53 (pp. 91-93).
Section 1.4: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 29, 31, 33 (pp. 102-103).

**Written Assignment**
Section 1.3: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 44, 54 (pp. 91-93).
Section 1.4: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 30, 32, 34 (pp. 102-103).

**Checklist Lesson 4**

Chapter 1, Section 1.3

The derivative is the slope (at a given point)
The derivative of a constant function
The derivative of a linear function
The secant line calculation of the derivative

Chapter 1, Section 1.4

The limit of the difference quotient
Nondifferentiability
Factoring to obtain limits
Lesson 5
Continuity and Differentiability

Reading Assignments
Chapter 1: Section 1.5, pp. 104-108; Section 1.6, pp. 110-114.

Introduction
In this lesson the important concept of continuity and its relationship to the differentiability of a function is discussed. In the second part of the lesson, additional rules for computing derivatives are presented.

Comments
It is important that you develop an intuitive idea of the concepts of continuity and differentiability. In other words, you should be able to determine the differentiability and continuity of a function at a point by examining the graph of the function. Also, you should know the limit criteria for continuity and be able to apply it.

Differentiability of a function $f(x)$ at $x = a$ is defined in terms of a limit. If this limit does not exist then we say $f(x)$ is nondifferentiable at $x = a$. Geometrically, nondifferentiability manifests itself in several different ways.

![Graphs showing differentiability and nondifferentiability](image)

The graph of $y = f(x)$ has no tangent line here at this *sharp* cusp.  

The tangent line at $x = a$ is vertical. The slope of such a line is undefined. Therefore, the derivative at $x = a$ does not exist.

The next topic discussed is that of continuity. A function $f(x)$ is continuous at $x = a$ provided that its graph has no breaks or gaps as it passes through the point $[a, f(a)]$. In terms of limits, this can be stated as follows: function $f(x)$ is continuous at $x = a$ provided that $\lim_{x \to a} f(x) = f(a)$.

Carefully study example 3 on page 113 and practice problems 1 and 2 on page 114.
The notions of differentiability and continuity are related as follows:

- If a function \( f(x) \) is differentiable at \( x = a \), then \( f(x) \) is continuous at \( x = a \).
- If a function is continuous at \( x = a \), it may or may not be differentiable at \( x = a \).
- If a function is not continuous at \( x = a \), then it is not differentiable at \( x = a \).
- Polynomial functions are continuous and differentiable everywhere.
- Rational functions are continuous and differentiable everywhere they are defined.

In Section 1.6, three additional rules of differentiation greatly extend the number of functions we can differentiate.

**Constant Multiple Rule**

\[
\frac{d}{dx} \left[ k \cdot f(x) \right] = k \cdot \frac{d}{dx} \left[ f(x) \right], \text{ } k \text{ is constant}
\]

**Sum Rule**

\[
\frac{d}{dx} \left[ f(x) + g(x) \right] = \frac{d}{dx} \left[ f(x) \right] + \frac{d}{dx} \left[ g(x) \right]
\]

**General Power Rule**

\[
\frac{d}{dx} \left[ (g(x))^r \right] = r \cdot g(x)^{r-1} \cdot \frac{d}{dx} \left[ g(x) \right], \text{ } r \text{ is any real number}
\]

Study the examples in this section carefully.

**Practice Problems**

Section 1.5: 1, 3, 5, 7, 9, 11, 13, 15, 17 (pp. 108-110).
Section 1.6: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43 (pp. 114-116).

**Written Assignment**

Section 1.5: 2, 4, 6, 8, 10, 12, 14, 16, 18 (pp. 108-110).
Section 1.6: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 38, 40, 42, 44 (pp. 114-116).

**Checklist Lesson 5**

Chapter 1, Section 1.5

- Continuous at \( x = a \)
- Differentiable at \( x = a \)
Chapter 1, Section 1.6

**Power Rule**
\[ \frac{d}{dx}(x^r) = rx^{r-1} \]

**General Power Rule**
\[ \frac{d}{dx}[g(x)]^r = r[g(x)]^{r-1} \cdot \frac{d}{dx}[g(x)], \text{ } r \text{ is any real number} \]

**Constant Multiple Rule**
\[ \frac{d}{dx}[k \cdot f(x)] = k \frac{d}{dx}[f(x)], \text{ } k \text{ is constant} \]

**Sum Rule**
\[ \frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}[f(x)] + \frac{d}{dx}[g(x)] \]
Lesson 6
Derivatives, Continued

Reading Assignments
Chapter 1, Section 1.7, pp. 117-120; section 1.8, pp. 121-128.

Introduction
In this lesson additional concepts involving derivatives are developed. Different notation conventions for derivatives, along with the concept of higher order derivatives, are presented in the first section of this lesson. In the second section, a very important interpretation of the derivative—the rate of change of a function—is discussed.

Comments
Study the examples in section 1.7 carefully. You should be thoroughly familiar with the table at the top of page 119.

Section 1.8 examines an important interpretation of the derivative. The derivative \( f'(a) \) measures the rate of change of \( f(t) \) with respect to \( t \) at the point \( t = a \). Study the practice problem on page 128. This will help you determine if you are interpreting \( f'(t) \) and \( f(t) \) correctly. Also be familiar with the following ideas:

1. The marginal concept in economics (see page 127).
2. Velocity and acceleration.

Practice Problems
Section 1.7: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 33 (pp. 120-121).
Section 1.8: 1, 3, 7, 9, 23, 25, 29 (pp. 129-133).

Written Assignment
Section 1.7: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 34 (pp. 120-121).
Section 1.8: 2, 4, 8, 10, 24, 26 (pp. 129-133).

Checklist Lesson 6
Chapter 1, Section 1.7

Notation for first and second derivative

Chapter 1, Section 1.8

The derivative \( f'(a) \) measures the rate of change of \( f(t) \) with respect to \( t \) at \( t = a \)
\[
f(a + h) - f(a) \approx f'(a) \cdot h
\]
Marginal concept in economics
Position, velocity, and acceleration functions
Lesson 7

_Self-Study:_ Review for Exam 1

Do not submit this lesson for grading. After carefully working through the following questions, check your answers with the solutions found in the textbook.

**Reading Assignments**
Review assignments from chapter 0 and chapter 1, both practice and written. Review checklists at the end of each lesson.

**Comments**
You will need to know how to compute the derivative of a function using the definition of the derivative. You will be asked to do this on this examination and on your final examination.

For this self-study assignment, problems are selected from the supplementary exercises at the end of each chapter.

**Self-Study Assignments**
- **Chapter 0:** 1, 3, 5, 9, 11, 13, 15, 17, 25, 31, 33, 39, 41, 43 (pp. 59-60).
- **Chapter 1:** 1, 3, 5, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 51, 53, 59, 67, 77, 79 (pp. 134-136).
Exam 1 Information

It is time for you to make arrangements with your proctor to take Exam 1.

Prior to taking this exam:
- You must submit lessons 2-6 to your instructor before taking this exam.
- Please do not take this exam until you have received lessons 2-6 back from your instructor.
- Do not submit any subsequent lessons until you have taken this exam.

Exam components:
- This exam covers lessons 1-7 and is comprehensive.
- This is a CLOSED book and note exam.
- Use of a Texas Instrument TI-30X IIS 2-Line Scientific Calculator is ALLOWED. No other calculators allowed for test use. (See image to right.)
- This exam is worth 100 points.
- You will have two hours to take this exam.

Items to take with you when you take the exam:
- photo identification;
- V number.

Exam grades and comments:
- Graded exams will not be returned to you.
Lesson 8

Self-Study: Derivatives and Graphs of Functions

Do not submit this lesson for grading. After carefully working through the following questions, check your answers with the solutions found in the textbook.

Reading Assignments
Chapter 2, Section 2.1, pp. 137-144; Section 2.2, pp. 149-154.

Introduction
In this chapter the derivative is used to investigate several geometric properties of functions, and this information is used to systematically sketch the graphs of functions. In the last three sections of this chapter, optimization problems are solved.

In section 2.1 the basic concepts needed in discussing the graphs of functions are introduced.

In section 2.2 the properties of functions are determined by analyzing the first and second derivatives of the function.

Comments
Have a thorough understanding of the following:

- A function \( f(x) \) is increasing at \( x = a \) if \( f'(a) > 0 \) (See the first derivative rule.)
- A function \( f(x) \) is decreasing at \( x = a \) if \( f'(a) < 0 \) (See the first derivative rule.)
- Relative extreme points of a function, i.e., relative minimum point, relative maximum point
- Maximum value of a function
- Minimum value of a function
- The concavity of a function, i.e., concave up, concave down
- Inflection points of a function
- Intercepts \((x\text{ and } y)\)
- Undefined points, asymptotes

In the second section of this lesson, the first and second derivatives are used to determine the properties of functions.

The first derivative rule:

- If \( f'(a) > 0 \), then \( f(x) \) is increasing at \( x = a \).
- If \( f'(a) < 0 \), then \( f(x) \) is decreasing at \( x = a \).
- If \( f'(a) = 0 \), then \( f(x) \) may be increasing, decreasing, or neither at \( x = a \).

The second derivative rule:

- If \( f''(a) > 0 \) then \( f(x) \) is concave up at \( x = a \).
- If \( f''(a) < 0 \) then \( f(x) \) is concave down at \( x = a \).
- If \( f''(a) = 0 \) then \( f(x) \) may be concave up, concave down, or neither at \( x = a \).
**Special Note:** From this lesson onward, you will be expected to justify all claims of maximum or minimum by using the second derivative (see examples 1 and 2 in section 2.3).

**Self-Study Assignments**
Section 2.1: 1, 3, 5, 7, 13, 21, 35 (pp. 145-149).
Section 2.2: 1, 3, 5, 7, 9, 11, 13, 15, 17, 25, 27, 29 (pp. 155-158).

**Checklist Lesson 8**
Chapter 2, Section 2.1

- Increasing, decreasing functions
- Maximum value, minimum value
- Relative extreme point, relative minimum point, relative maximum point
- Concave up, concave down
- Inflection point
- $x$-intercept, $y$-intercept
- Asymptote

Chapter 2, Section 2.2

- First derivative rule
- Second derivative rule
Lesson 9
Sketching the Graphs of Functions

Reading Assignments
Chapter 2, Section 2.3, pp. 159-164; Section 2.4, pp. 166-171.

Introduction
In this lesson a systematic approach to sketching the graphs of functions is developed.

In the first section a technique for determining relative extreme points is introduced. Also, inflection points are found. In the second section of this lesson, all the techniques are summarized, including a look at some more involved problems.

Comments
Study the examples in section 2.3 carefully. The material discussed in section 2.3 can be summarized as follows: To locate relative extreme points, set \( f'(x) = 0 \) and solve for \( x \). Then apply the following test:

Second Derivative Test
1. Suppose \( f''(a) < 0 \).
2. Then if \( f''(a) > 0 \), \( f(x) \) is concave up at \( x = a \) and \( f(x) \) has a relative minimum at \( x = a \).
3. If \( f''(a) < 0 \), \( f(x) \) is concave down at \( x = a \) and \( f(x) \) has a relative maximum at \( x = a \).
4. If \( f''(a) = 0 \), then we can draw no conclusions. This last situation is dealt with in the next section.

To determine inflection points, first find all points where \( f''(x) = 0 \). Suppose \( f''(a) = 0 \), then check the concavity on either side of \( x = a \). If the concavity changes, then \( f(x) \) has an inflection point at \( x = a \). Study the examples in section 2.4 carefully.

Example: Find all relative maxima and minima of \( f(x) = 4x^5 + 5x^4 - 1 \).

\[
\begin{align*}
f'(x) &= 20x^4 + 20x^3 \\
f''(x) &= 80x^3 + 60x^2 \\
\text{Set } f'(x) &= 0 \text{ and solve for } x \\
20x^3 + 20x^2 &= 0 \\
20x^2(x + 1) &= 0 \\
x &= 0, x = -1, f(0) = 1 \text{ and } f(1) = 0, \text{ plot the points } (0, -1) (-1, 0) \\
f''(-1) &= 80(-1)^3 + 60(-1)^2 < 0, \text{ so } f(x) \text{ is concave down, and has a relative maximum at } x = -1 \\
f''(0) &= 0. \text{ No conclusion can be drawn.}
\end{align*}
\]

Now apply the first derivative test to see if \( x = 0 \) is a relative extreme point. Check points to the left and right of \( x = 0 \). (Here left and right mean left and right on the number line.) Then check to the left (or to the right), but only go as far as the next value of \( x \) where \( f'(x) = 0 \).

To check to the left of \( x = 0 \), i.e., if \(-1 < x < 0\), \( f'(x) < 0 \); (choose any value of \( x \) in this interval and compute \( f'(x) \) there.)
Suppose \( x = -\frac{1}{2} \): \( f'(-\frac{1}{2}) = 20(-\frac{1}{2})^4 + 20(-\frac{1}{2})^3 < 0 \).

Now we find that for each \( x > 0 \), \( f'(x) > 0 \); (again, choose any value for \( x > 0 \), say \( x = 1 \); then compute \( f'(1) > 0 \). At \( x = 0 \), \( f(x) \) changes from decreasing to increasing, so \( f(x) \) has a relative minimum at \( x = 0 \). The graph is sketched to the right.

In example 3, Section 2.4, on page 169, the author sketches the graph of \( f(x) = (x - 2)^4 - 1 \). Although this analysis is perfectly valid, it does not illustrate the method of determining relative extreme points as illustrated in the Summary of Curve-Sketching Techniques. You might choose to employ the same analysis as in the example previously illustrated. Here is a summary of the author’s analysis.

\[
\begin{align*}
  f'(x) &= 4(x - 2)^3 \\
  f''(x) &= 12(x - 2)^2 \\
  f'(2) &= 0 \\
  f''(2) &= 0 \text{ (we cannot use the second derivative rule)}
\end{align*}
\]

Check the first derivative on either side of \( x = 2 \).
\( f'(x) = 4(x - 2)^3 \).
If \( x < 2 \), \( f'(x) < 0 \) and thus \( f(x) \) is decreasing on \((-\infty, 2)\).
If \( x > 2 \), \( f'(x) > 0 \) and thus \( f(x) \) is increasing on \((2, \infty)\).
Therefore, \( f(x) \) has a relative minimum at \( x = 2 \).

**Practice Problems**

Section 2.3: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 23, 27, 29 (pp. 164-166).
Section 2.4: 1, 3, 5, 7, 9, 13, 19, 21 (pp. 171-172).

**Written Assignment**

Section 2.3: 2, 8, 10, 12, 16, 18, 20, 24, 30 (pp. 164-166).
Section 2.4: 4, 6, 8, 10, 18, 24 (pp. 171-172).

**Checklist Lesson 9**

Sections 2.3 and 2.4

- How to look for possible relative extreme points
- How to decide whether a relative extreme point is a relative maximum or minimum
- How to look for possible inflection points
- Summary of curve-sketching techniques
Lesson 10
Optimization Problems

Reading Assignments
Chapter 2, Section 2.5, pp. 173-179.

Introduction
This lesson begins the study of optimization problems.

Comments
Study the examples carefully. When working problems, follow the guidelines listed on page 179. Carefully read each problem before attempting to solve it.

One of the most frequently asked questions is, How do I know which variable to solve for in the constraint equation? Generally it makes no difference; however, in some cases a great deal of unpleasant work may be avoided by carefully examining the constraint and objective equations. This is illustrated in the discussion of example 5 below.

In example 3, on page 175 and example 4, on page 176, it makes no difference which variable we solve for. Example 3 shows $2x + w = 40$ and $A = wx$. Either $x$ or $w$ can be solved for easily in the constraint equation. Also, since $x$, and $w$ appear as factors in the objective equation, there is no particular advantage in writing the objective equation in terms of one particular variable (see problem 25 on page 181). See a similar situation in example 4.

In example 5, we have the following constraint and objective equations $\lambda + 2 \pi r = 84$, and $V = \pi r^2 \lambda$. Notice that in the objective equation, $\lambda$ appears as a factor (note: it is just raised to the first power) and $\lambda$ can easily be solved for in the constraint equation. It is just as possible to solve for $r$ in the constraint equation giving:

$$r = \frac{84 - \lambda}{2\pi}$$

(an unpleasant-looking fraction). Then substituting:

$$V(\lambda) = \pi \left( \frac{84 - \lambda}{2\pi} \right)^2 \lambda.$$ 

Solve the problem using this equation. Simplifying:

$$V(\lambda) = \pi \left( \frac{7056}{168} + \frac{\lambda}{4\pi} - \frac{\lambda^2}{16} \right).$$

$$V'(\lambda) = \frac{1}{4\pi} (7056 - 336\lambda + 3\lambda^2)$$

$$V''(\lambda) = \frac{1}{4\pi} (-336 + 6\lambda).$$

Set:

$$V'(\lambda) = 0$$

and solve for $\lambda$. 

27
\[
\frac{1}{4\pi} (7056 - 3361 + \rho^2) = 0
\]

\[
2352 - 1121 + \rho^2 = 0
\]

\[
(1 - 28)(1 + 84) = 0
\]

\(1 = 28\) or \(1 = -84\) (\textit{reject} \(1 = -84\) as length cannot be negative)

The problem can be finished by using the second derivative test and sketching the graph of \(V(x)\). (Try to do this for practice.) Therefore, we see that example 5 can be solved either way. In general, try to keep the objective function as simple as possible.

**Practice Problems**
Section 2.5: 1, 3, 7, 9, 11, 13, 15, 17, 19 (pp. 180-182).

**Written Assignment**
Section 2.5: 2, 4, 6, 10, 12, 14, 16, 18, 24 (pp. 180-182).

**Checklist Lesson 10**
Chapter 2, Section 2.5

- Objective equation
- Constraint equation
- Suggestions for solving an optimization problem (page 179)
Lesson 11
Further Optimization Problems

Reading Assignments
Chapter 2, Section 2.6, pp. 183-189.

Introduction
The study of optimization problems is continued and different types of problems are discussed.

Comments
The examples given in the text and the solved problems in the supplement should provide you with all the help needed.

Two different types of problems are introduced here, a revenue type problem and the inventory problem. Also study example 5. Notice in this problem that the function has its maximum value at endpoint \( x = 0 \).

Practice Problems
Section 2.6: 1, 3, 5, 7, 11, 13, 15, 17, 19, 21 (pp. 189-192).

Written Assignment
Section 2.6: 2, 4, 6, 8, 12, 14, 16, 18, 20, 22 (pp. 189-192).

Checklist Lesson 11
Same as lesson 10.
Lesson 12
Applications of Calculus

Reading Assignments
Chapter 2, Section 2.7, pp. 193-201.

Introduction
This lesson concludes the discussion of applied problems that seek to maximize or minimize quantities. In this section the optimization problems are those that arise in business and economics.

Comments
No new methods are involved here. The techniques developed in solving optimization problems are now applied to problems that arise in business and economics. Several new functions are defined here; you should become familiar with them.

Practice Problems
Section 2.7: 1, 4, 5, 7, 9, 11, 17 (pp. 201-204).

Written Assignment
Section 2.7: 2, 3, 6, 8, 10, 12, 18 (pp. 201-204).

Checklist Lesson 12
Chapter 2, Section 2.7

Cost function
Revenue function
Marginal cost function
Marginal revenue function
Profit function
Lesson 13
Differentiation Techniques

Reading Assignments
Chapter 3: Section 3.1, pp. 209-214; Section 3.2, pp. 218-221.

Introduction
In this chapter differentiation techniques that apply to products and quotients are developed. Also, the differentiation formula for computing the derivative of the composition of functions is presented in its most general form.

Comments
In section 3.1 of this chapter, the following formulas are given:

Product Rule
\[ \frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x) \]

Quotient Rule
\[ \frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2} \]

As stated before, do not be concerned with memorizing the proofs of these rules; however, read the proofs carefully and convince yourself that they prove the validity of the formulas. Study the examples presented in this section thoroughly.

The next section (3.2) discusses one of the most important rules in differential calculus, the chain rule. The term chain rule refers to functions that are composed as a chain, or a composition of other functions. A special case of this rule—the general power rule—has already been studied.

Chain Rule
\[ \frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x) \]

Read through this section again, study the proof of the chain rule carefully and be convinced that it does prove the validity of the formula.
The alternate way of stating the chain rule is as follows:

1. Given \( y = f(g(x)) \), set \( u = g(x) \) and \( y = f(u) \).

2. This notation shows \( \frac{du}{dx} = g'(x) \), and \( \frac{dy}{du} = f'(u) = f'(g(x)) \).

3. Then by the chain rule we have \( \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \).

**Practice Problems**
Section 3.1: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33 (pp. 214-217).
Section 3.2: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 25, 29, 37, 41 (pp. 221-224).

**Written Assignment**
Section 3.1: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34 (pp. 214-217).
Section 3.2: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 30, 32, 38, 42 (pp. 221-224).

**Checklist Lesson 13**
Chapter 3, Section 3.1

**Product Rule:**

\[
\frac{d}{dx} \left[ f(x)g(x) \right] = f(x)g'(x) + g(x)f'(x)
\]

**Quotient Rule:**

\[
\frac{d}{dx} \left[ \frac{f(x)}{g(x)} \right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}
\]

Chapter 3, Section 3.2

**Chain Rule**

\[
\frac{d}{dx} \left[ f[g(x)] \right] = f'(g(x)) \cdot g'(x)
\]
Lesson 14
Self-Study: Review for Exam 2

Do not submit this lesson for grading. After carefully working through the following questions, check your answers with solutions found in the textbook.

Reading Assignments
Review assignments from chapter 2 and chapter 3 (both practice and written assignments). Review checklists at the end of each lesson.

Comments
You will be asked to sketch the graph of a function (see chapter 2, sections 2.3 and 2.4). Also, three optimization problems will be given on this examination. One problem each will come from sections 2.5, 2.6, and 2.7 in chapter 2.

Practice Problems
Chapter 2: 13, 17, 19, 23, 27, 33, 35, 55, 58, 59 (pp. 205-208).
Chapter 3: 1, 3, 5, 7, 9, 11, 17 (pp. 234-237).
Exam 2 Information

It is time for you to make arrangements with your proctor to take Exam 2.

Prior to taking this exam:
- You must submit lessons 9-13 to your instructor before taking this exam.
- Please do not take this exam until you have received lessons 9-13 back from your instructor.
- Do not submit any subsequent lessons until you have taken this exam.

Exam components:
- This exam covers lessons 8-14 and is comprehensive.
- This is a CLOSED book and note exam.
- Use of a Texas Instrument TI-30X IIS 2-Line Scientific Calculator is ALLOWED. No other calculators allowed for test use. (See image to right.)
- This exam is worth 100 points.
- You will have two hours to take this exam.

Items to take with you when you take the exam:
- photo identification;
- V number;

Exam grades and comments:
- Graded exams will not be returned to you.
Lesson 15
Transcendental Functions

Reading Assignments
Chapter 4, Section 4.1, pp. 238-241; Section 4.2 pp. 242-245.

Introduction
So far in the course, all the functions that have been studied have been algebraic in nature, that is, they are functions formed from polynomial functions by the basic operations of addition, subtraction, multiplication, division, and taking of rational powers and roots. This chapter introduces two important so-called transcendental functions (functions that cannot be formed from polynomials as described above). Examples of functions of this type are exponential and logarithmic functions.

This lesson first introduces the general exponential function, reviews properties of exponents, and then in the second part of this lesson the exponential function using base $e$ is examined.

Comments
Review the properties of exponents again. You should be familiar with the general exponential function $f(x) = b^x$. Be sure to study example 2, section 4.1, on page 241.

Practice Problems
Section 4.1: 1, 3, 5, 17, 19, 21, 23, 25, 27, 29 (pp. 241-242).
Section 4.2: 5, 7, 9, 11, 13, 15, 17, 19, 21, 23 (pp. 245-247).

Written Assignment
Section 4.1: 2, 4, 6, 18, 20, 22, 24, 26, 28, 30 (pp. 241-242).
Section 4.2: 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24 (pp. 245-247).

Checklist Lesson 15
Chapter 4, Section 4.1

\begin{align*}
b^x \cdot b^y &= b^{x+y} \\
b^{-x} &= \frac{1}{b^x} \\
b^x \div b^y &= b^{x-y} \\
(b^x)^y &= b^{xy}
\end{align*}
Chapter 4, Section 4.2

\[ a^x b^x = (ab)^x \]

\[ \frac{a^x}{b^x} = \left( \frac{a}{b} \right)^x \]

\[ b^0 = 1 \]

Definition of \( e \) and \( e^x \) (see page 242)
Lesson 16
Differentiation of Exponential Functions

Reading Assignments
Chapter 4, Section 4.3, pp. 247-250.

Introduction
In this lesson differentiation formulas for exponential functions are derived and the subject of differential equations is introduced.

Comments
Using the chain rule along with the derivative formula \( \frac{d}{dx}(e^x) = e^x \) from section 4.2 illustrates the following.

\[
\frac{d}{dx} e^{g(x)} = e^{g(x)} g'(x)
\]

Students frequently have trouble applying this formula. Review examples 1, 2, and 3 on page 248; also practice problem 1 before proceeding with the assignment. A special case of the above rule is the following:

\[
\frac{d}{dx} e^{kx} = e^{kx} \cdot k = ke^{kx}
\]

This provides an introduction to the study of differential equations. Carefully follow the discussion on page 249. Also be familiar with the general shape of the graph of a function \( y = e^{kx} \) where \( k \) is positive, and where \( k \) is negative. Note that for any positive number \( b \), there is a number \( k \) such that \( b = e^k \). Thus \( b^x = (e^k)^x = e^{kx} \).

Practice Problems
Section 4.3: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 37, 39, 41, 43 (pp. 251-252).

Written Assignment
Section 4.3: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 38, 40, 42, 44 (pp. 251-252).

Checklist Lesson 16
Chapter 4, Section 4.3

\[
\frac{d}{dx} e^{kx} = ke^{kx}
\]

\[
\frac{d}{dx} e^{g(x)} = e^{g(x)} g'(x)
\]

Graph of \( e^{kx} \)
If \( y = f(x) \) satisfies \( y' = ky \), then \( y = ce^{kx} \) for some constant \( c \).
Lesson 17
The Natural Logarithm Function

Reading Assignments
Chapter 4, Section 4.4, pp. 253-256; Section 4.5, pp. 258-260.

Introduction
In this lesson the natural logarithm function is constructed as the inverse of the exponential function $e^x$, and differentiation formulas for this function are derived.

Comments
The most important concepts in section 4.4 are the relationship between the exponential function $e^x$, and the natural logarithmic function $\ln x$: For $x > 0$, $e^{\ln x} = x$. The natural logarithm function and the exponential function are inverses of one another. The relationship between $e^x$ and $\ln x$ may be used to solve equations. Study examples 1 and 2 on page 255 carefully.

The following natural logarithms appear frequently. Know their values without having to use a calculator.

\[
\begin{align*}
\ln 1 &= 0 \\
\ln e &= 1 \\
\ln \left(\frac{1}{e}\right) &= \ln e^{-1} = -1 \\
\ln e &= 1
\end{align*}
\]

Also be familiar with the graph of $\ln x$.

In section 4.5, the formula for computing the derivative of the natural logarithm function is derived. Students frequently use this formula incorrectly. Study example 1, on page 258 carefully.

\[
\frac{d}{dx} [\ln g(x)] = \frac{1}{g(x)} \cdot \frac{d}{dx} g(x)
\]

\[
= \frac{1}{g(x)} \cdot g'(x)
\]

\[
= \frac{g'(x)}{g(x)}
\]

Reading example 3 carefully, the following is demonstrated.

\[
\frac{d}{dx} \ln |x| = \frac{1}{x}, \quad x \neq 0
\]

Practice Problems
Section 4.4: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 35 (pp. 256-257).
Section 4.5: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29 (pp. 260-261).
Written Assignment
Section 4.4: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 36 (pp. 256-257).
Section 4.5: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30 (pp. 260-261).

Checklist Lesson 17
Chapter 4, Section 4.4

Reflection over the line \( y = x \) (graphs of \( \ln x \) and \( e^x \))
Definition of \( \ln x \)
\( \ln 1 = 0 \)
\( \ln e = 1 \)

Chapter 4, Section 4.5
\[
\frac{d}{dx} \ln x = \frac{1}{x}, \quad x \neq 0,
\]
\[
\frac{d}{dx} [\ln g(x)] = \frac{g'(x)}{g(x)}
\]
Lesson 18
Properties of the Natural Logarithm Function

Reading Assignments
Chapter 4, Section 4.6, pp. 262-264.

Introduction
In this lesson, properties of logarithms are reviewed and used in simplifying the differentiation of some logarithmic functions.

Comments
Properties of the natural logarithm function. Let $x$ and $y$ be positive numbers and $b$ be any number.

1. $\ln(xy) = \ln x + \ln y$
2. $\ln \left(\frac{1}{x}\right) = \ln 1 - \ln x = -\ln x$
3. $\ln \left(\frac{x}{y}\right) = \ln x - \ln y$
4. $\ln x^b = b \ln x$

Practice Problems
Section 4.6: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35 (p. 265).

Written Assignment
Section 4.6: 2, 4, 6, 8, 10, 12, 14, 20, 22, 24, 36, 38 (pp. 265).

Checklist Lesson 18
Chapter 4, Section 4.6

\[
\begin{align*}
\ln(xy) &= \ln x + \ln y \\
\ln \left(\frac{1}{x}\right) &= -\ln x \\
\ln \left(\frac{x}{y}\right) &= \ln x - \ln y \\
\ln (x^b) &= b \ln x
\end{align*}
\]
Lesson 19
Exponential Growth and Decay

Reading Assignments
Chapter 5, Section 5.1, pp. 268-275.

Introduction
In this chapter several applications of exponential functions are developed. In this lesson the law of exponential growth (decay) is introduced and several applications are discussed.

Comments
As presented in the last chapter the following are shown:

- The function $y = ce^{kt}$ satisfies the differential equation $y' = ky$.
- Conversely, if $y = f(t)$ satisfies the differential equation above, then $y = ce^{kt}$ for some constant $c$.
- If $f(t) = ce^{kt}$, then $c$ is the value of $f(t)$ at time $t = 0$.

Exponential Growth
If at every instant the rate of increase of the quantity is proportional to the quantity at that instant, it is said that the quantity is growing exponentially or exhibiting exponential growth. Use $P(t)$ to denote the amount of the quantity present at time $t$. The differential equation $y' = ky$ can be written as $P'(t) = kP(t)$. The solution to this differential equation then has the form $P(t) = P_0e^{kt}$, where $P_0$ is the initial amount (the quantity present at time $t = 0$) and $k$ is the growth constant.

Exponential Decay or Negative Exponential Growth
It is known that, at any instant, the rate at which a radioactive substance is decaying is proportional to the amount of substance that has not yet disintegrated. If $P(t)$ is the quantity present at time $t$, the $P'(t)$ is the rate of decay. $P'(t)$ must be negative since $P(t)$ is decreasing. Thus write $P'(t) = kP(t)$ for some negative constant $k$.

To emphasize the fact that the proportionality constant is negative, replace $k$ by $-\lambda$, where $\lambda$ is a positive constant. Then $P(t)$ satisfies the differential equation $P'(t) = -\lambda P(t)$. The general solution of the differential equation has the form $P(t) = P_0e^{-\lambda t}$. Such a function is called an exponential decay function. The constant $\lambda$ is called the decay constant. As an example, suppose that $P(t) = P_0e^{-\lambda t}$. The decay constant $\lambda = .01$, not -.01. Radiocarbon dating is an interesting application of radioactive decay. Read the discussion on page 273 carefully.
Practice Problems
Section 5.1: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 25 (pp. 275-278).

Written Assignment
Section 5.1: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22 (pp. 275-278).

Checklist Lesson 19
Chapter 5, Section 5.1

\[ y' = ky \text{ has the solution } y = ce^{kt} \]
Half-life of a radioactive element
Exponential growth
Exponential decay
Lesson 20

Self-Study: Compound Interest

Do not submit this lesson for grading. After carefully working through the following questions, check your answers with the solutions found in the textbook.

Reading Assignments
Chapter 5, Section 5.2, pp. 279-282.

Introduction
This lesson extends the application of exponential functions to the growth of capital at a fixed interest rate.

Comments
This section begins with a discussion of compound interest. The amount \( A \) present after \( t \) years is given by the formula

\[
A = P \left( 1 + \frac{r}{m} \right)^{mt}
\]

where \( P \) is the principal amount,
\( r \) is the interest rate per annum (i.e., per year),
\( m \) is the number of interest periods per year, and
\( t \) is the number of years.

See examples 5 and 6 on pages 44-45. These examples discuss the effective annual interest rate, a term that is frequently used by savings institutions.

If interest is compounded continuously, then \( A = Pe^{rt} \), where \( A \) is the amount after \( t \) years, \( P \) is the principal, and \( r \) is the interest rate per annum.

Present Value
To receive an amount \( A \) in \( t \) years, invested at an interest rate \( r \), compounded continuously, then the amount \( P \), the principal that is to be invested today, is called the present value of the amount \( A \). Begin with the formula

\[
A = Pe^{rt} \quad \text{and solve for } P
\]

then

\[
\frac{A}{e^{rt}} = P \quad \text{or}
\]

\[
Ae^{-rt} = P
\]

The concept of present value is important in business and economics.

Self-Study Assignments
Section 5.2: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23 (pp. 283-284).
Checklist Lesson 20
Chapter 5, Section 5.2

Compound interest, \( A = P \left( 1 + \frac{r}{m} \right)^{mt} \)

Continuous compounding of interest, \( A = Pe^{rt} \)

Present Value
Lesson 21
Self-Study: Review for Exam 3

Do not submit this lesson for grading. After carefully working through the following questions, check your answers with the solutions found in the textbook.

Reading Assignments
Review assignments from chapter 4 and chapter 5 (both practice and written assignments). Review checklists at the end of each lesson.

Comments
Although no graphing appears on this examination, you will be asked to find the relative extreme points of a function (see page 251, problems 27-32 for examples).

Practice Problems
Chapter 4: 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61, 63, 91, 93 (pp. 266-267).
Chapter 5: 1, 3, 5, 13 (pp. 305-307).
Exam 3 Information

It is time for you to make arrangements with your proctor to take Exam3.

Prior to taking this exam:
• You must submit lessons 15-19 to your instructor before taking this exam.
• Please do not take this exam until you have received lessons 15-19 back from your instructor.
• Do not submit any subsequent lessons until you have taken this exam.

Exam components:
• This exam covers lessons 15-21 and is comprehensive.
• This is a CLOSED book and note exam.
• Use of a Texas Instrument TI-30X IIS 2-Line Scientific Calculator is ALLOWED. No other calculators allowed for test use. (See image to right.)
• This exam is worth 100 points.
• You will have two hours to take this exam.

Items to take with you when you take the exam:
• photo identification;
• V number;
• non-graphing calculator;

Exam grades and comments:
• Graded exams will not be returned to you.
Lesson 22
Antidifferentiation

Reading Assignments
Chapter 6, Section 6.1, pp. 308-314.

Introduction
Having completed a study of the derivative and its applications, it is now time for the second most important topic of calculus, the integral.

The definite integral is defined as a limit of sums (6.2 will not be covered).

The Fundamental Theorem of Calculus (6.3) gives a simple direct method of evaluating the definite integral by using the process of antidifferentiation. In this lesson the process of antidifferentiation is discussed and several formulas for computing antiderivatives are presented.

Comments
By now it is possible to solve a great many problems of the form, given a function \( f(x) \), find \( f'(x) \).

This section deals with the reverse process. Given a function \( f(x) \), find a function \( F(x) \) such that \( F'(x) = f(x) \). This function \( F(x) \) is called the antiderivative of \( f(x) \). The standard way to express this fact is to write

\[
\int f(x) \, dx = F(x) + c
\]

The symbol \( \int \) is called the integral sign. The entire notation \( \int f(x) \, dx \) is called the indefinite integral and stands for the antidifferentiation of the function \( f(x) \). Record the variable of interest by prefacing it with the letter \( d \). For example, if the variable of interest is \( t \) rather than \( x \), write \( \int f(t) \, dt \) or the antiderivative of \( f(t) \). If \( F(x) \) is an antiderivative of \( f(x) \), then so is \( F(x) + c \) for any constant \( c \). It turns out that all the antiderivatives of \( f(x) \) must be in the form \( F(x) + c \).

This result may be stated as follows:

- If \( F_1(x) \) and \( F_2(x) \) are two antiderivatives of the same function \( f(x) \), then \( F_1(x) \) and \( F_2(x) \) differ by a constant.

Also, the following result is sometimes useful:

- If \( f'(x) = 0 \) for all \( x \), then \( F(x) = c \) for some constant \( c \).
By reversing differentiation rules, the following integration formulas are found:

\[ \int x^r \, dx = \frac{1}{r+1} x^{r+1} + c, \quad r \neq -1 \]

\[ \int e^{kx} \, dx = \frac{1}{k} e^{kx} + c, \quad k \neq 0 \]

\[ \int \frac{1}{x} \, dx = \ln |x| + c, \quad x \neq 0 \]

*Note:* This last formula may appear as:

\[ \int x^{-1} \, dx = \ln |x| + c \]

Also, it is possible to obtain the following rules for antiderivatives:

\[ \int [f(x) + g(x)] \, dx = \int f(x) \, dx + \int g(x) \, dx \]

\[ \int kf(x) \, dx = k \int f(x) \, dx, \quad \text{where } k \text{ is a constant} \]

### Practice Problems

**Section 6.1:** 1, 3, 5, 7, 9, 11, 21, 23, 25, 27, 29, 31, 33, 35, 37, 45, 49 (pp. 315-317).

### Written Assignment

**Section 6.1:** 2, 4, 6, 8, 10, 12, 22, 24, 26, 28, 32, 34, 36, 38, 46, 48 (pp. 315-317).

### Checklist Lesson 22

**Chapter 6, Section 6.1**

- **Indefinite Integral**
  - \[ \int \frac{1}{x} \, dx = \ln |x| + c, \quad x \neq 0 \]
  - \[ \int kf(x) \, dx = k \int f(x) \, dx, \quad \text{where } k \text{ is constant} \]
  - \[ \int x^r \, dx = \frac{1}{r+1} x^{r+1} + c, \quad r \neq -1 \]
  - \[ \int e^{kx} \, dx = \frac{1}{k} e^{kx} + c, \quad k \neq 0 \]
  - \[ \int [f(x) + g(x)] \, dx = \int f(x) \, dx + \int g(x) \, dx \]
Lesson 23
Definite Integrals and the Fundamental Theorem

Reading Assignments
Chapter 6, Section 6.3, pp. 327-336.

Introduction
This lesson introduces the definite integral. The Fundamental Theorem of Calculus provides a way to compute the definite integral.

Comments
In this chapter the definite integral is defined first for a non-negative function (the graph of the function does not go below the x-axis, but the graph may touch the x-axis).

The definite integral of $f(x)$ over the interval from $x = a$ to $x = b$ is defined as the area of the region under the graph of $y = f(x)$ and above the x-axis from $x = a$ to $x = b$. The definite integral of $f(x)$ is denoted by

$$\int_{a}^{b} f(x) \, dx.$$ 

In general the graph of $y = f(x)$ determines regions above and below the x-axis. In this case the definite integral of $f(x)$ is defined as the total area of the regions above the x-axis minus the total area of the regions below the x-axis. Example: $\int_{a}^{b} f(x) \, dx = [\text{area of } A + C] - [\text{area of } B]$
In the definite integral \( \int_a^b f(x)dx \)

1. \( a \) is called the lower limit of integration,
2. \( b \) is called the upper limit of integration,
3. and the function \( f(x) \) is called the integrand.

It is important to distinguish between the definite integral, which is a number, and the indefinite integral, which represents all antiderivatives of \( f(x) \). The following theorem connects the two key concepts of calculus.

**The Fundamental Theorem of Calculus:**

1. Let \( f(x) \) be a continuous function for \( a \leq x \leq b \), and let \( F(x) \) be an antiderivative of \( f(x) \).
2. Then \( \int_a^b f(x)dx = F(b) - F(a) \).
3. The quantity \( F(b) - F(a) \) is called the net change of \( F(x) \) from \( x = a \) to \( x = b \). It is abbreviated by the symbol \( F(x) \bigg|_a^b \).

As an example of net change of a function, examine the following.

- For a particular ball thrown into the air, its height (in feet) above the ground after \( t \) seconds is given by \( s(t) = -16t^2 + 128t + 5 \). The net change in \( s(t) \), i.e., height of the ball from \( t = 1 \) to \( t = 2 \) seconds, is written \( \left(-16t^2 + 128t + 5\right) \bigg|_1^2 \) and is evaluated as \( s(2) - s(1) \).

Study example 4 on page 330 carefully. Note that \( c \) is usually omitted when working out definite integrals.

In evaluating definite integrals, it may be helpful to view the process in two steps:

1. First find an antiderivative of the function \( f(x) \).
2. Second, compute \( F(b) - F(a) \), i.e. \( \int_a^b f(x)dx = F(x) \bigg|_a^b \)

**Example:**

\[
\int_1^2 (3x^2 + 2x - 1)dx = \left[x^3 + x^2 - x\right]_1^2 \\
= \left[2^3 + (2)^2 - 2\right] - \left[1^3 + (-1)^2 - (-1)\right] \\
= \left[8 + 4 - 2\right] - \left[1 + 1 + 1\right] \\
= \left[10\right] - \left[3\right] \\
= 9
\]

Study the solution to this example and example 6 on page 331 carefully. The calculations show how to use parentheses and brackets to avoid arithmetic errors. In evaluating a definite integral, just apply the Fundamental Theorem of Calculus. The answer may be a positive number, a negative number, or zero.
The curve only represents an area if the function \( f(x) \geq 0 \) for \( a \leq x \leq b \), (i.e. if \( f(x) \) is non-negative on the interval \( a \leq x \leq b \).

Here are some examples.

Evaluate \( \int_{-1}^{1} x \, dx \).

\[
\int_{-1}^{1} x \, dx = \frac{x^2}{2} \bigg|_{-1}^{1} = \frac{1^2}{2} - \frac{(-1)^2}{2} = \frac{1}{2} - \frac{1}{2} = 0
\]

Recall that if the curve goes below the \( x \)-axis, the integral is equal to the area above the \( x \)-axis minus the area below the \( x \)-axis.

Evaluate \( \int_{-1}^{2} x^3 \, dx = \frac{x^4}{4} \bigg|_{-1}^{2} = \frac{2^4}{4} - \left[-\frac{(-1)^3}{4}\right] = \frac{16}{4} - \frac{1}{4} = \frac{15}{4} \)

The number above is equal to the [area of region A] – [area of region B].
Evaluate \[ \int_{-1}^{1} (x^2 - 2) \, dx = \left[ \frac{x^3}{3} - 2x \right]_{-1}^{1} \]

\[ \begin{aligned}
&= \left( \frac{1}{3} \right) \left( 1^2 - 2 \right) - \left( -1 \right) - 2(1) \\
&= \left[ \frac{3}{3} \right] - \left[ \frac{3}{3} \right] \\
&= 1 - 1 + 2 \\
&= 1 - 2 + 1 - 2 \\
&= 3 - 3 \\
&= 2 - 4 \\
&= \frac{2}{3} - \frac{12}{3} \\
&= \frac{-10}{3}
\end{aligned} \]

In this example the area above the x-axis is zero, so the number \(-\frac{10}{3}\) is the negative of the area of region A.

**Practice Problems**

Section 6.3: 1, 3, 5, 7, 9, 11, 13, 15, 23, 25, 27, 29, 31, 39, 41 (pp. 336-338).

**Written Assignment**

Section 6.3: 2, 4, 6, 8, 10, 12, 14, 24, 26, 28, 30, 32, 34, 40, 42, 44 (pp. 336-338).

**Checklist Lesson 23**

Chapter 6, Section 6.3

Net change: \[ F(x) \bigg|_{a}^{b} \]

**Fundamental Theorem of Calculus**

\[ \int_{a}^{b} f(x) \, dx = F(b) - F(a), F'(x) = f(x) \]
Lesson 24
Self-Study: Areas in the xy-Plane

Do not submit this lesson for grading. After carefully working through the following questions, check your answers with the solutions found in the textbook.

Reading Assignments
Chapter 6, Section 6.4, pp. 339-345.

Introduction
In this section the definite integral is used to compute the area of a region between two curves, and important properties of the definite integral are stated.

Comments
Important properties of the definite integral are: Let $f(x)$ and $g(x)$ be functions and $a$, $b$, and $k$ be any constants, then:

1. \[ \int_a^b f(x)dx + \int_a^b g(x)dx = \int_a^b [f(x) + g(x)]dx \]
2. \[ \int_a^b f(x)dx - \int_a^b g(x)dx = \int_a^b [f(x) - g(x)]dx \]
3. \[ \int_a^b kf(x)dx = k\int_a^b f(x)dx \]

The important result from this section is the following:

- If $y = f(x)$ lies above $y = g(x)$ from $x = a$ to $x = b$, the area of the region between $f(x)$ and $g(x)$ from $x = a$ to $x = b$ is given by $\int_a^b [f(x) - g(x)]dx$.

Study examples 1, 2, 3, 4, and 5 very carefully, and then study the example on the following page.
Example: Find the area between the curve \( y = x^2 - 4 \) and the \( x \)-axis from \( x = -1 \) to \( x = 1 \). View this as finding the area between the curves \( y = x^2 - 4 \) and \( y = 0 \) (\( x \)-axis) from \( x = -1 \) to \( x = 1 \).

Sketch the curve and note \( 0 > x^2 - 4 \) for \(-1 \leq x \leq 1\).

\[
\text{Area } \int_{-1}^{1} [0 - (x^2 - 4)] \, dx = \int_{1}^{1} (-x^2 + 4) \, dx
\]

\[
= \left[ -\frac{x^3}{3} + 4x \right]_{1}^{1}
\]

\[
= \left( -\frac{1}{3} + 4 \right) - \left( -\frac{1}{3} + 4 \right) = 4(-1)
\]

\[
= -4 + 4 = 0
\]

\[
= \int_{-1}^{1} (x^2 - 4) \, dx
\]

\[
= 22 - \frac{22}{3}
\]

\[
= \frac{22}{3}
\]

\[
\int_{-1}^{1} (x^2 - 4) \, dx \text{ does not give an area here since between -1 and 1, the curve } y = x^2 - 4 \text{ lies below the } x \text{-axis. Note: } \int_{-1}^{1} (x^2 - 4) \, dx = -\frac{22}{3}.
\]

**Self-Study Assignment**

Section 6.4: 1, 3, 5, 7, 9, 11, 13, 17, 21 (pp. 346-347).

**Checklist Lesson 24**

Chapter 6, Section 6.4

Area between two curves: \( \int_{a}^{b} [f(x) - g(x)] \, dx \)
Lesson 25
Techniques of Integration

Reading Assignments
Chapter 6, Section 6.6, pp. 358-365.

Introduction
This lesson introduces the student to an integration technique known as substitution. This is the integral version of the chain rule.

Comments
Integration by Substitution

As the authors state, knowing the correct substitution to make is a skill that develops through practice. Basically, look for an occurrence of function composition \( f(g(x)) \) where function \( f(x) \) is easily integrated and \( g'(x) \) also appears in the integral. Sometimes \( g'(x) \) does not appear exactly as needed but can be obtained by multiplying by a constant.

Frequently the two areas that cause difficulty are:

1. making the correct substitution and rewriting the integral in terms of a different variable, and
2. integrating the new integral.

This lesson tackles the second problem first. If the proper substitution has been made, the new integral will have the form that is already familiar. Yet simply because the integral involves a different variable, students seem to find it difficult. First review the previously developed integration formulas.

\[
\int x^r \, dx = \frac{1}{r+1} x^{r+1} + c, \quad r \neq -1
\]

\[
\int e^{kx} \, dx = \frac{1}{k} e^{kx} + c, \quad k \neq 0
\]

\[
\int \frac{1}{x} \, dx = \ln|x| + c, \quad x \neq 0
\]

These formulas could have been written in terms of any variable, such as \( t \).

\[
\int t^r \, dt = \frac{1}{r+1} t^{r+1} + c, \quad r \neq -1
\]

\[
\int e^{kt} \, dt = \frac{1}{k} e^{kt} + c, \quad k \neq 0
\]

\[
\int \frac{1}{t} \, dt = \ln|t| + c, \quad t \neq 0
\]
Here are some problems to consider:

1. \[ \int u^4 \, du = \frac{u^5}{4} + c \]

2. \[ \int \frac{1}{u} \, du = \int u^{-1} \, du = \ln|u| + c \]

3. \[ \int e^u \, du = -e^u + c \]

4. \[ \int \frac{1}{4}u^{10} \, du = \frac{1}{4} \int u^{10} \, du = \frac{1}{4} \left( \frac{u^{11}}{11} \right) + c = \frac{u^{11}}{44} + c \]

Next we shall develop some guidelines to help us. Recall the chain rule: \[ \frac{d}{dx} [f(g(x))] = f'(g(x)) \cdot g'(x). \]

The integral version of the chain rule is \[ \int f(g(x)) \cdot g'(x) \, dx = \int f(u) \, du, \text{ where } u = g(x). \]

To make a substitution that will simplify the integral, identify \( g(x) \). Examine the uses of the chain rule in the differentiation formulas used previously.

**Generalized Power Rule.**

A. \[ \frac{d}{dx} \left( \frac{g(x)^r}{r} \right) = r \cdot g(x)^{r-1} \cdot g'(x) \]

**Exponential Function**

B. \[ \frac{d}{dx} e^{g(x)} = e^{g(x)} \cdot g'(x) \]

**Natural Logarithm Function**

C. \[ \frac{d}{dx} \ln g(x) = \frac{1}{g(x)} \cdot g'(x) = \frac{g'(x)}{g(x)} = [g(x)]^{-1} \cdot g'(x) \]

Notice in A and C that the function \( g(x) \) can be identified as the function of \( x \) raised to a power. In B, \( g(x) \) is the exponent of the exponential function.

First look at the integral. If it is written as a quotient, write it as a product using negative exponents, i.e., \[ \int \frac{2x}{x^2 + 1} \, dx. \] The integrand is \[ \frac{2x}{x^2 + 1}. \] Write it as \[ 2x \left( x^2 + 1 \right)^{-1}. \]

Next, look in the integrand for a function of \( x \) raised to a power. (This may appear as a quantity enclosed in parentheses raised to a power.) In the above example, the quantity \( x^2 + 1 \) is raised to the -1 power.

Finally, look at the remaining factors of the integrand. Are the remaining factors close to being the derivative of the function \( x \) previously found? (By close we mean differing by a constant multiple.)
The above example identified \( x^2 + 1 \) as being a function of \( x \) raised to a power. The remaining factor is \( 2x \), which is the derivative of \( x^2 + 1 \) exactly.

Thus if substituting \( u = x^2 + 1 \), then \( du = 2dx \). Hence, \( \int 2x(\text{x}^2 + 1)'dx = \int u'\,du \), a much simpler-looking integral. To finish \( \int u'\,du = \ln|u| + c = \ln|x^2 + 1| + c \).

Suppose a function of \( x \) raised to a power was not found. Look for a function of \( x \) appearing in the exponent of the exponential function, i.e. \( \int 2xe^{x^2}\,dx \).

Let \( u = x^2 \). Look at the remaining factors of the integrand. Are the factors close to being the derivative of this expression? The remaining factors in the above integrand are the derivative of this function.

Therefore, let \( u = x^2 \), then \( du = 2dx \). Thus \( \int 2xe^{x^2}\,dx = \int e^u\,du \) is a much simpler integral. To finish
\[
\int e^u\,du = e^u + c = e^{x^2} + c.
\]

**A word of caution:** Look at the following integral.

\[
\int (2x-5)(\text{x}^2-5x+3)\,dx
\]

Here two quantities enclosed in parentheses are raised to the same power, positive 1. How is the substitution made here? Compare the two quantities:

\((2x-5)\) and \((\text{x}^2-5x+3)\)

Is one of these quantities close to being the derivative of the other? Note that the quantity on the left is the derivative of the one on the right.

Therefore let \( u = x^2 - 5x + 3 \)
\[
du = (2x - 5)\,dx
\]

Then
\[
\int (2x-5)(\text{x}^2-5x+3)\,dx = \int u\,du = \frac{\text{u}^2}{2} + c = \frac{(\text{x}^2 - 5x + 3)^2}{2} + c
\]

Is there another way to compute this integral? Multiply \((2x - 5)(\text{x}^2 - 5x + 3)\) to get a third-degree polynomial, then integrate term by term. But this is usually a more difficult practice. The following examples should answer any remaining questions.

Determine \( \int 10x^3(x^4 + 10)'\,dx \)

Let \( u = x^4 + 10 \)
then \( du = 4x^3\,dx \)

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In this example we have the *wrong* constant; we need $4x^3$, not $10x^3$. Write the integral in the following way.

$$\int 10x^3(x^4 + 10)^4 \, dx = 10 \int x^3(x^4 + 10)^4 \, dx$$

Now use the play on numbers introduced in the text, $(1/4) \cdot 4 = 1$.

$$10 \int x^3(x^4 + 10)^4 \, dx = 10 \cdot \frac{1}{4} \int 4x^3(x^4 + 10)^4 \, dx$$

$$= \frac{10}{4} \left[ u^4 \right] + c$$

$$= \frac{1}{2} u^5 + c$$

$$= \frac{1}{2} (x^4 + 10)^5 + c$$

The following is a challenging problem. Try to solve it first.

$$\int \frac{1}{x \ln 5x} \, dx = \int \frac{1}{x} \cdot \frac{1}{\ln 5x} \, dx$$

$$= \int \left[ \frac{5x}{\ln 5x} \right] \, dx$$

Let $u = \ln 5x$

then $du = \frac{5}{5x} \cdot 5 \, dx = \frac{1}{x} \, dx$

So,

$$\int \frac{1}{\ln 5x} \, dx = \int u^{-1} \, du$$

$$= \ln |u| + c$$

$$= \ln |\ln 5x| + c$$
**Practice Problems**
Section 6.6: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 33, 37, 41 (pp. 365-366).

**Written Assignment**
Section 6.6: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 32, 36, 44 (pp. 365-366).

**Checklist Lesson 25**
Chapter 6, Section 6.6

\[ \int f(g(x)) \cdot g'(x) \, dx = \int f(u) \, du, \text{ where } u = g(x) \]
Lesson 26
Partial Derivatives

Reading Assignments
Chapter 7, Section 7.1, pp. 385-389; Section 7.2, pp. 392-399.

Introduction
In the preceding part of this course, differential and integral calculus for functions of one variable were developed. In this chapter, the concept of a function of several variables is introduced and the concept of differentiation is extended to these functions.

Comments
Up to this point in the course, the discussions have been restricted to functions of one variable. The techniques used in analyzing functions of one variable extend to functions of several variables. Section 7.1 introduces functions of several variables (primarily two variables). In Section 7.2, partial derivatives are introduced. No new differentiation techniques are needed. Compute the derivative with respect to one variable at a time, treating the other variables as constants.

Practice Problems
Section 7.1: 1, 3, 5, 7, 9, 11, 15 (pp. 390-391).
Section 7.2: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25 (pp. 399-401).

Written Assignment
Section 7.1: 2, 4, 6, 8, 10, 12, 16, 22 (pp. 390-391).
Section 7.2: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26 (pp. 399-401).

Checklist Lesson 26
Chapter 7, Section 7.1

Level curves
\[
\frac{\partial f}{\partial x}(a, b), \quad \frac{\partial f}{\partial y}(a, b)
\]

Chapter 7, Section 7.2

Partial derivatives
Second partial derivatives
Lesson 27
Relative Maxima and Relative Minima of Functions of Several Variables

Reading Assignments
Chapter 7, Section 7.3, pp. 401-407; Section 7.4, pp. 410-418.

Introduction
In this lesson the concepts involving relative maxima and relative minima developed in the study of functions of one variable are now extended to functions of several variables (primarily two variables). Also in this lesson, a very powerful technique for solving constrained optimization problems is introduced. It is called the method of Lagrange multipliers.

Comments
The notion of a relative extreme point of a function of two variables parallels the notion of a relative extreme point for a function of one variable.

o If \( f(x, y) \) is a function of two variables, it is said that \( f(x, y) \) has a relative maximum when \( x = a \) and \( y = b \) if \( f(x, y) \) is at most equal to \( f(a, b) \) whenever \( x \) is near \( a \) and \( y \) is near \( b \).

o Similarly, \( f(x, y) \) is said to have a relative minimum when \( x = a \) and \( y = b \) if \( f(x, y) \) is at least equal to \( f(a, b) \) whenever \( x \) is near \( a \) and \( y \) is near \( b \). (See figure 1 on page 402.)

Also, the notion of absolute maximum value and absolute minimum value can be extended to functions of two (or more) variables. \( f(x, y) \) has an absolute maximum value at \( x = a \) and \( y = b \) if \( f(a, b) \) is greater than or equal to \( f(x, y) \) for any \( (x, y) \) in the domain of the function. Similarly, \( f(x, y) \) has an absolute minimum value at \( x = a \) and \( y = b \) if \( f(a, b) \) is less than or equal to \( f(x, y) \) for any \( (x, y) \) in the domain of the function.

It is not surprising then that for \( f(x, y) \) to have a relative extreme at \((a, b)\) a necessary condition is that

\[
\frac{\partial f}{\partial x}(a, b) = 0 \quad \text{and} \quad \frac{\partial f}{\partial y}(a, b) = 0.
\]

Thus the procedure for finding a relative extrema of a function of two variables will be similar to finding relative extrema of a function of one variable. First find all the points where the function might possibly have a relative extreme point by setting both partial derivatives equal to zero and solving the system of equations that results.

As in the case of one variable, the conditions just discussed for the partial derivatives of \( f(x, y) \) are not sufficient for the point \((a, b)\) to be a relative extremum. There is an analog to the second derivative test for functions of one variable. It is a great deal more complicated.

Second derivative test for functions of two variables
Suppose that \( f(x, y) \) is a function of two variables and \((a, b)\) is a point such that \( \frac{\partial f}{\partial x}(a, b) = 0 \) and \( \frac{\partial f}{\partial y}(a, b) = 0 \) and let

\[
D(x, y) = \frac{\partial^2 f}{\partial x^2} \cdot \frac{\partial^2 f}{\partial y^2} - \left( \frac{\partial^2 f}{\partial x \partial y} \right)^2.
\]
1. If \( D(a, b) > 0 \) and \( \frac{\partial^2 f}{\partial x^2}(a, b) > 0 \), then \( f(x, y) \) has a relative minimum at \( (a, b) \).

2. If \( D(a, b) > 0 \) and \( \frac{\partial^2 f}{\partial x^2}(a, b) < 0 \), then \( f(x, y) \) has a relative maximum at \( (a, b) \).

3. If \( D(a, b) < 0 \) then \( f(x, y) \) has neither a relative maximum nor a relative minimum at \( (a, b) \).

4. If \( D(a, b) = 0 \), then no conclusion can be drawn from this test.

As in the case of functions of one variable, the relative maximum or minimum points of a function may not be the points where the function achieves its maximum or minimum values. As the authors point out, the examples and exercises have been chosen so that if an absolute maximum or minimum value of \( f(x, y) \) exists, it will occur at a point where \( f(x, y) \) has a relative maximum or minimum.

The case of a function of three or more variables can be handled in a similar fashion. For a function of three variables, say \( f(x, y, z) \), possible relative extreme points are found by determining all points \( (a, b, c) \) where

\[
\begin{align*}
\frac{\partial f}{\partial x}(a, b, c) &= 0 \\
\frac{\partial f}{\partial y}(a, b, c) &= 0 \\
\frac{\partial f}{\partial z}(a, b, c) &= 0
\end{align*}
\]

In section 7.4, a powerful technique for solving constrained optimization problems is introduced—the Lagrange multiplier. In many applied problems a function of two variables, \( f(x, y) \), is to be optimized subject to a restriction or constraint on the variables. This restriction or constraint is usually stated in terms of an equation, \( g(x, y) = 0 \). The basic idea of the method introduced here is to define an auxiliary function of three variables \( F(x, y, \lambda) \) as \( F(x, y, \lambda) = f(x, y) + \lambda g(x, y) \).

Lamda \( \lambda \) is called the Lagrange multiplier. Locate all the points \( (x, y, \lambda) \) where the partial derivatives of \( F(x, y, \lambda) \) are equal to zero; then among the corresponding points \( (x, y) \) it is possible to find all points where \( f(x, y) \) may have constrained relative maxima or minima.

As the authors state, in most applications it is known that an absolute maximum or minimum exists. In the event that the method of Lagrange multipliers produces exactly one possible relative extreme value, it is assumed that is indeed the sought-after absolute extreme value. (This is certainly not always the case but it will be in the problems in this text.)

Study example 1 on pages 412-413 carefully. Note the technique used in solving the three equations in three unknowns.

**Practice Problems**

Section 7.3: 1, 3, 5, 7, 9, 17, 19, 21, 23, 25, 27, 29 (pp. 408-409).

Section 7.4: 1, 3 (pp. 418-420)
Written Assignment
Section 7.3:  2, 4, 6, 8, 10, 18, 20, 22, 24, 26, 28, 30, 32, 34 (pp. 408-409).
Section 7.4:  2, 4 (pp. 418-420).

Checklist Lesson 27

Chapter 7, Section 7.3

Relative maxima and minima of functions of several variables
First derivative test
Second derivative test in two variables

Chapter 7, Section 7.4

Method of Lagrange multiplier
Objective function
Constraint equation
Lesson 28
Self-Study: Review for Exam 4

Do not submit this lesson for grading. After carefully working through
the following questions, check your answers with the solutions found in the textbook.

Reading Assignments
Review assignments from chapter 6 and chapter 7 (both practice and written).
Review lesson checklists at the end of each lesson.

Comments
This self-study lesson is a review for the fourth and last two-hour examination. By now you are familiar
with the checklists and use of supplementary exercises as a means of review.

Substantial portions of these chapters have been omitted from our study. In general, these sections tend to
deal with specific uses of calculus. Though you are not responsible for these sections, you may find them
interesting.

On this examination you will be asked to solve a problem using the method of Lagrange multiplier. You
will also be asked to find the relative extreme points of a function of two variables. The formula for the
function $D(x, y)$ will be given to you; you are expected to know everything else about the second
derivative test.

Practice Problems
Chapter 6: 1, 3, 7, 9, 10, 13, 27, 37, 39 (pp. 380-382).
Chapter 7: 1, 3, 5, 7, 11, 13, 17, 21, 25 (pp. 446-447).
Exam 4 Information

It is time for you to make arrangements with your proctor to take Exam 4.

Prior to taking this exam:
- You must submit lessons 22, 23, 25-27 to your instructor before taking this exam.
- Please do not take this exam until you have received lessons 22, 23, 25-27 back from your instructor.
- Do not submit any subsequent lessons until you have taken this exam.

Exam components:
- This exam covers lessons 22-28; it will contain questions on only the material covered since exam 3.
- This is a CLOSED book and note exam.
- Use of a Texas Instrument TI-30X IIS 2-Line Scientific Calculator is ALLOWED. No other calculators allowed for test use. (See image to right.)
- This exam is worth 100 points.
- You will have two hours to take this exam.

Items to take with you when you take the exam:
- photo identification;
- V number;
- non-graphing calculator;
- Graded exams will not be returned to you.
Final Exam Information

It is time for you to make arrangements with your proctor to take the Final Exam.

Prior to taking this exam:
- You must have taken Exam 4 before taking this exam.
- Please do not take this exam until you have received Exam 4 comments back from your instructor.
- Do not submit any subsequent lessons until you have taken this exam.

Exam components:
- This exam covers lessons 3-28. There will not be direct questions concerning lessons 1 and 2 (chapter 0 in the text.) In addition to the mathematical concepts taught in the course, you will be expected to know the business and economic concepts used in the application-type problems. In particular, know those concepts involving revenue, cost, profit, inventory control, and linear demand.
- The final will not differ significantly from your two-hour examinations in the type of questions asked, except for the fact that it is comprehensive and will require three hours to complete. The problems are roughly 80 percent theoretical (compute derivatives, integrals, and so on) and 20 percent application (optimization problems, etc.).
- This is a CLOSED book and note exam.
- Use of a Texas Instrument TI-30X IIS 2-Line Scientific Calculator is ALLOWED. No other calculators allowed for test use. (See image to right.)
- This exam is worth 200 points.
- You will have three hours to take this exam.

Items to take with you when you take the exam:
- photo identification;
- V number;
- non-graphing calculator.

Exam grades and comments:
- Graded exams will not be returned to you.