Math 144
Analytic Trigonometry

The University of Idaho in statewide cooperation with Boise State University — Idaho State University — Lewis-Clark State College
Math 144
Analytic Trigonometry

University of Idaho
1 Semester-Hour Credits

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Math 144: Analytic Trigonometry 1 Semester-Hour Credits: UI

Welcome!
Whether you are a new or returning student, welcome to the Independent Study in Idaho (ISI) program. Below, you will find information pertinent to your course including the course description, course materials, course objectives, as well as information about assignments, exams, and grading. If you have any questions or concerns, please contact the ISI office for clarification before beginning your course.

Policies and Procedures
Refer to the ISI website at www.uidaho.edu/isi and select Students for the most current policies and procedures, including information on setting up accounts, student confidentiality, exams, proctors, transcripts, course exchanges, refunds, academic integrity, library resources, and disability support and other services.

Course Description
Trigonometric functions, inverse functions, applications.
Prerequisite: Sufficient score on SAT, ACT, or math placement test. Students may qualify by enrolling concurrently in Math 143 or Math 170. Required test scores can be found at http://www.uidaho.edu/registrar/registration/placement.
UI students: Not open for credit to students who have previous high school or college credit in trigonometry. Polya Math Center unavailable for ISI students.

Required: Internet access
Recommended: headphones

16 graded assignments, 3 proctored exams

Students may submit up to 1 unit at a time/1 unit per week. A unit is a set of lessons preceding an exam. For example, lessons 1A through 6 comprise unit 1. Note that completing 1 chapter per week is a very demanding pace. It is more realistic to complete two lessons per week and prepare for each test over the course of a few days before taking it. Before taking exams, students MUST wait for grades and feedback on assignments, which may take up to three weeks after date of receipt by the instructor.

ALL assignments and exams must be submitted to receive a final grade for the course.

Course Materials
Required Course Materials
- Trigsted, Kirk, MyMathLab for Math 144 (MyLabsPlus Access Code), Pearson, 9781256698180. Purchase a Math 144 access code from the VandalStore (University of Idaho bookstore).
- Math 144 Course Guide available on BbLearn upon course registration
Course Delivery
All ISI courses are delivered through BbLearn, an online management system that hosts the course lessons and assignments and other items that are essential to the course. Upon registration, the student will receive a Registration Confirmation Email with information on how to access ISI courses online.

Course Goals
The primary purpose of Trigonometry is to improve your skills and competency in trigonometry to prepare you for calculus. Another goal is to help you develop your mathematical learning skills so that you will be more confident in future mathematical courses.

Course Objectives
After completing Math 144, the student should be able to do the following without the use of a calculator:

- Understand the right triangle definitions of the trigonometric functions
- Understand the unit circle definitions of the trigonometric functions
- Evaluate trigonometric functions of angles belonging to the $\frac{\pi}{3}$, $\frac{\pi}{4}$, and $\frac{\pi}{6}$ families
- Sketch the graph of functions of the form $y = A\sin(Bx+C)+D$ and $y = A\cos(Bx+C)+D$
- Understand the graph of the tangent function and its properties
- Understand the graph of the cosecant and secant functions and their properties
- Understand the graphs of the inverse sine function, inverse cosine function, and inverse tangent function
- Evaluate expressions involving inverse trigonometric functions
- Verify trigonometric identities
- Solve trigonometric equations

Lessons
Overview
This course will cover three chapters (Ch 1, Ch 2, and Ch 3). There are a total of 16 homework assignments (one written and 15 online). Your homework score will be computed at the end of the course. All assignments will be averaged and the average will be scaled to 10% of your total grade. You can think of the average as being scaled to 10 points.

Each lesson may include the following components:
- lesson objectives
- reading assignments
- important terms
- lecture
- written assignment, project, or activity
Study Hints:

- The best way to be successful in this course is to complete all of the assigned work. Carefully read the eText and watch the videos and animations before attempting the corresponding homework problems. Take notes in the space provided on the lesson pages and complete the homework when indicated.
- Carefully copy down and work out every homework problem. Keep a notebook of the homework problems.
- Keep a copy of every assignment submitted.
- Complete all reading assignments.
- Set a schedule allowing for course completion one month prior to your personal deadline. An Assignment Submission Log is provided for this purpose.
- Web pages and URL links in the World Wide Web are continuously changing. Contact your instructor if you find a broken Web page or URL.

Refer to the Course Rules in BbLearn for further details on assignment requirements and submission.

Exams

- The exams are cumulative.
- Calculators are not allowed during exams.
- No notes of any kind are allowed during tests, although a sheet of formulas will be provided during Exam 3.
- NO DEVICES which are capable of transmitting or receiving data, including but not limited to watches, phones, tablets, iPods, and calculators, may be on your person during the exam. Any such items are expected to be left at home or left with the proctor. Failure to do so will result in a zero on the exam and possibly a failing grade for the course.
- You must wait for grades and comments on assignments prior to taking subsequent exams.
- For your instructor’s exam guidelines, refer to the Course Rules in BbLearn.
- Refer to Grading for specific information on assignment/exam points and percentages.

There will be one exam after the completion of each chapter (see the Notebook Table of Contents for a list of sections covered) for a total of three exams throughout the semester. Each exam is worth 100 points. Each exam will cover material from the entire course leading up to the exam. Therefore, exam 3 will cover the entire course. Students may take only one exam per day.

Proctor Selection/Scheduling Exams

All exams require a proctor unless an exam is self-administered. To submit your Proctor Information Form online, visit the ISI website and select Forms, Proctor Information Form. Submit this form at least two weeks before your first exam. Refer to Students, Assignments and Exams on the ISI website for information on acceptable and unacceptable proctors.
Grading
The course grade will be based upon the following considerations:

<table>
<thead>
<tr>
<th>Assignment</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homework</td>
<td>10%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Exam</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exam 1</td>
<td>30%</td>
</tr>
<tr>
<td>Exam 2</td>
<td>30%</td>
</tr>
<tr>
<td>Exam 3</td>
<td>30%</td>
</tr>
<tr>
<td>Total</td>
<td>90%</td>
</tr>
</tbody>
</table>

Your course grade will be based on the percentage that you have earned as follows:
90% guarantees an A
80% guarantees a B
70% guarantees a C
60% guarantees a D

The final course grade is issued after all assignments and exams have been graded.

Academic Honesty
Acts of academic dishonesty, including cheating or plagiarism are considered a very serious transgression and may result in a grade of F for the course.

Students are expected to maintain Academic Honesty in all their work. Collaboration is encouraged on homework assignments. All tests are considered individual work and must be completed without unauthorized assistance of any kind, including the help of other students, tutors, notes, or calculators. All test materials and scratch paper are to be turned in with the test paper and attempting to bring test work out of the testing area and/or share that work with other students is considered cheating.

The University of Idaho has defined acceptable behavior in the Student Code of Conduct Article II.A-1 – Academic Dishonesty [rev. 7-98, 7-05, 7-14, ed. 7-09]. The following summarizes relevant points related to your math course:

- **Because academic honesty and integrity are core values at a university, the faculty finds that even one incident of academic dishonesty may merit expulsion.**
- **Cheating on classroom or outside assignments, examinations, or tests is a violation of this code.**
- Plagiarism, falsification of academic records, falsification of records and the acquisition or use of test materials without faculty authorization are considered forms of academic dishonesty and, as such, are violations of this code.
• Instructors and students are responsible for maintaining academic standards and integrity in their classes. Consequences for academic dishonesty may be imposed by the course instructor. Such consequences may include but cannot exceed a grade of "F" in the course.

The full text of the Student Code of Conduct may be found at http://www.uidaho.edu/DOS/judicialaffairs/studentcodeofconduct/Student%20Code%20of%20Conduct

About the Course Developer
Theresa Allen has degrees in chemistry, math, and chemical engineering. She has been teaching math at the University of Idaho since 1999. Her teaching experience includes intermediate algebra, precalculus, trigonometry, finite math, introductory Calculus, and math for liberal arts majors.

Dr. Allen hopes that every student in this course will learn something new about the subject and about him/herself as a learner. The subject matter appears in nature and man-made contexts in exciting ways. She looks forward to students finding examples of the topic in the wider world.

Contacting Your Instructor
Instructor contact information is posted on your BbLearn site under Course Rules.
# Math 144 Grade Summary

Enter your scores in the charts below, as percentages. Remember that you may only take one test in a day. See the Syllabus for the requirements.

## Homework (HW)

<table>
<thead>
<tr>
<th>Homework</th>
<th>Possible</th>
<th>Earned</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduction</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>Section 1.1</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>Section 1.3</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>Section 1.4</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>Section 1.5</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>Section 1.6</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>Section 2.1A</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>Section 2.1B</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>Section 2.2</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>Section 2.3</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>Section 2.4</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>Section 2.5</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>Section 3.1</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>Section 3.2</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>Section 3.3</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>Section 3.5</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>1600</strong></td>
<td></td>
</tr>
</tbody>
</table>

Divide the HW total by 160 and write the result here: ___________  Put this result in the table below:

## Grading Scale

<table>
<thead>
<tr>
<th>Grade</th>
<th>Range</th>
<th>Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>90 – 100 % (pts)</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>80 – 89% (pts)</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>70 – 79% (pts)</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>60 – 69% (pts)</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>Below 50% (pts)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>TOTALS</strong></th>
<th><strong>Possible</strong></th>
<th><strong>Earned</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>HW (average of all 16)</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Test 1</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>Test 2</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>Test 3</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>Sum of the scores above</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>
Math 144 Grade Summary Example

This person earned an “A” for the course with very reasonable scores.

### Homework (HW)

<table>
<thead>
<tr>
<th>Homework</th>
<th>Possible</th>
<th>Earned</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduction</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Section 1.1</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Section 1.3</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Section 1.4</td>
<td>100</td>
<td>95</td>
</tr>
<tr>
<td>Section 1.5</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Section 1.6</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Section 2.1A</td>
<td>100</td>
<td>80</td>
</tr>
<tr>
<td>Section 2.1B</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Section 2.2</td>
<td>100</td>
<td>99</td>
</tr>
<tr>
<td>Section 2.3</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Section 2.4</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Section 2.5</td>
<td>100</td>
<td>95</td>
</tr>
<tr>
<td>Section 3.1</td>
<td>100</td>
<td>93</td>
</tr>
<tr>
<td>Section 3.2</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Section 3.3</td>
<td>100</td>
<td>95</td>
</tr>
<tr>
<td>Section 3.5</td>
<td>100</td>
<td>93</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>1600</strong></td>
<td><strong>1550</strong></td>
</tr>
</tbody>
</table>

Divide the HW total by 160 (this example has 1550/160) and write the result here: _9.7_. Put this result in the table below:

### Grading Scale

<table>
<thead>
<tr>
<th>Grade</th>
<th>Minimum – Maximum % (pts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>90 – 100 % (pts)</td>
</tr>
<tr>
<td>B</td>
<td>80 – 89% (pts)</td>
</tr>
<tr>
<td>C</td>
<td>70 – 79% (pts)</td>
</tr>
<tr>
<td>D</td>
<td>60 – 69% (pts)</td>
</tr>
<tr>
<td>F</td>
<td>Below 50% (pts)</td>
</tr>
</tbody>
</table>

### TOTALS

<table>
<thead>
<tr>
<th></th>
<th>Possible</th>
<th>Earned</th>
</tr>
</thead>
<tbody>
<tr>
<td>HW (average of all 16)</td>
<td>10</td>
<td>9.7</td>
</tr>
<tr>
<td>Test 1 (high score)</td>
<td>30</td>
<td>28.5</td>
</tr>
<tr>
<td>Test 2 (high score)</td>
<td>30</td>
<td>27.0</td>
</tr>
<tr>
<td>Test 3 (high score)</td>
<td>30</td>
<td>25.5</td>
</tr>
<tr>
<td>Sum of the scores above</td>
<td>100</td>
<td>90.7</td>
</tr>
</tbody>
</table>
Sample Lesson  
Chapter 1: Section 1.3

Lesson Objectives
- Classifying Triangles
- Using the Pythagorean Theorem
- Understanding Similar Triangles
- Understanding the Special Right Triangles
- Using Similar Triangles to Solve Applied Problems

Overview
This Lesson will cover Section 1.3 in your e-Text. Work through each of the following tasks, carefully filling out the following pages.

Section 1.3 Triangles
- Work through Objective 1 then do problems #1-2
- Work through Objective 2 then do problems #3-4
- Work through Objective 3 then do problems #5-8
- Work through Objective 4 then do problems #9-14
- Work through Objective 5 then do problems #15-16

Section 1.3 Triangles
Section 1.3 Objective 1 Classifying Triangles

What does it mean for two angles or sides of a triangle to be congruent?

What is an acute triangle?

What is an obtuse triangle?

What is a right triangle?

Sketch and label an acute, obtuse, and right triangle, as seen in Figure 18.
What is a **scalene triangle**?

What is an **isosceles triangle**?

What is an **equilateral triangle**?

Sketch a scalene, isosceles, and equilateral triangle, as seen in Figure 19.

Work through Example 1 showing all work below.

Classify the given triangle as acute, obtuse, right, scalene, isosceles, or equilateral. State all that apply.

Watch the animation located on page 1.3-5 of the eText and explain why every isosceles right triangle has two acute angles that have a measure of $\frac{\pi}{4}$ radians.

**WORK SECTION 1.3 HW EXERCISES #1-2**
Section 1.3 Objective 2 Using the Pythagorean Theorem

What is The Pythagorean Theorem? (Hint: See the text box on page 1.3-6.)

Work through Example 2 and show all work below.

Use the Pythagorean Theorem to find the length of the missing side of each of the given right triangles.

![Right triangles](image)

WORK SECTION 1.3 HW EXERCISES #3-4

Section 1.3 Objective 3 Understanding Similar Triangles

What is the definition of similar triangles?

What are the Properties of Similar Triangles?
Work through the video accompanying Example 4 showing all work below. Triangles ABC and XYZ are similar. Find the lengths of the missing sides of triangle ABC.

What is the definition of the Proportionality Constant of Similar Triangles?
Work through the animation accompanying Example 5 showing all work below. The triangles below are similar. Find the proportionality constant. Then find the lengths of the missing sides.

Work through the video accompanying Example 6 showing all work below. The right triangles below are similar. Determine the lengths of the missing sides.
WORK SECTION 1.3 HW EXERCISES #5-8

Section 1.3 Objective 4 Understanding the Special Right Triangles

Watch the animation on page 1.3-17 which describes the $\frac{\pi}{4}, \frac{\pi}{4}, \frac{\pi}{2}$ right triangle and take notes on how to establish a relationship between the lengths of the sides.

Sketch and label the $\frac{\pi}{4}, \frac{\pi}{4}, \frac{\pi}{2}$ right triangle as seen in Figure 24.
Watch the animation on page 1.3-18 which describes the \( \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2} \) right triangle and take notes on how to establish a relationship between the lengths of the sides.

Sketch and label the \( \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2} \) right triangle as seen in Figure 28.
Work through the interactive video with Example 7 and show all work below. Determine the lengths of the missing sides of each right triangle.

**WORK SECTION 1.3 HW EXERCISES #9-14**

**Section 1.3 Objective 5 Using Similar Triangles to Solve Applied Problems**

Work through Example 8 and show all work below.

The shadow of a cell tower is 80 feet long. A boy 3 feet 9 inches tall is standing next to the tower. If the boy’s shadow is 6 feet long, find the height of the cell tower.
Work through the video with Example 9 and show all work below.

Two people are standing on opposite sides of a small river. One person is located at point Q, a distance of 20 feet from a bridge. The other person is standing on the southeast corner of the bridge at point P. The angle between the bridge and the line of sight from P to Q is 30°. Use this information to determine the length of the bridge and the distance between the two people. Round your answer to two decimal places as needed. (Note that you will need a calculator for this exercise. Only applications that do not require a calculator will be on your tests!)

WORK SECTION 1.3 HW EXERCISES #15-16