Smarter Balanced Assessment Consortium:

Mathematics Item Specifications Grades 3-5

Developed by Measured Progress/ETS Collaborative
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Mathematics General Specifications
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Introduction

The Smarter Balanced Assessment Consortium (*Smarter Balanced*) is committed to using evidence-centered design (ECD) in its development of its assessment system. As part of this design, *Smarter Balanced* established four “Claims” regarding what students should know and be able to do in the domain of mathematics. The principles of ECD require that each of these four claims be accompanied by statements about the kinds of evidence that would be sufficient to support the claims. These evidence statements are articulated as “assessment targets.”

The initial work of using ECD to establish the four claims and the accompanying targets for mathematics has been completed by the consortium and is articulated in the document “Content Specifications for the Summative Assessment of the *Common Core State Standards for Mathematics* (CCSSM).” The content specifications document is a necessary complement to these test item and performance task specifications, and the documents should be used together to fully understand how to write the items and tasks needed to provide the evidence to support these claims and targets.

What follows is a guide to help item writers develop items/tasks that meet the requirements set forth by *Smarter Balanced*. This specification document will also provide useful information to other stakeholders concerning the types of items/tasks that will comprise the summative assessments beginning in 2015.

Assessing Mathematics

The Common Core State Standards for Mathematics require that mathematical content and mathematical practices be *connected* (CCSSM, p. 8). In addition, two of the major design principles of the standards are *focus* and *coherence* (CCSSM, p. 3). Together, these features of the standards have important implications for the design of the *Smarter Balanced* assessment system.

The next section will discuss the various types of items/tasks *Smarter Balanced* intends to use in its assessment system in order to connect the mathematics content with the mathematical practices defined in the CCSS. The following definitions are used for various parts of an item or task.

- Item: the entire item, including the stimulus, question/prompt, answer/options, scoring criteria, and metadata.
- Task: similar to an item, yet typically more involved and usually associated with constructed-response, extended-response, and performance tasks.
- Stimulus: the text, source (e.g., video clip), and/or graphic about which the item is written. The stimulus provides the context of the item/task to which the student must respond.
- Stem: the statement of the question or prompt to which the student responds.
- Options: the responses to a selected-response (SR) item from which the student selects one or more answers.
- Distractors: the incorrect response options to an SR item.
- Distractor Analysis: the item writer's analysis of the options or rationale for inclusion of specific options.
- Key: the correct response(s) to an item.
- Top-Score Response: one example of a complete and correct response to an item/task.
- Scoring Rubric: the descriptions for each score point for an item/task that is assigned more than one point for a correct response.

Stimulus for Mathematics Items/Tasks

Stimulus materials for mathematics items/tasks usually take the form of graphs, models, figures, etc. that students must read and examine in order to respond to the items/tasks. Below are some general guidelines for mathematics stimulus materials. For a comprehensive discussion of stimulus materials, refer to the specific section of the *Test Item and Performance Task Specifications* that is devoted entirely to Stimulus Specifications.

- Graphs, pictures, models, tables, figures, and any other graphic presented with an item or task must meet the art and style guidelines adopted by *Smarter Balanced*. When using graphics,
    - use visuals that mirror and parallel the wording and expectations of the accompanying text;
    - illustrate only necessary information in the graphics so as not to confuse and/or distract test takers; and
    - represent important parts of the item/task in visual images if the graphics serve to increase item access for more test takers.
- Use contexts that are both familiar and meaningful to the mathematics being assessed, as well as appropriate to the grade level of the test takers.
- Keep the amount of reading to a minimum and non-mathematics vocabulary should be at least one grade level below that of the test takers.
- Include reliable source and verification information with the item/task when referencing factual information and using real-world data.

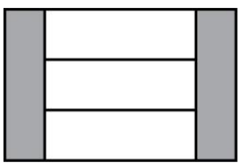
Selected-Response Items

Traditionally, selected-response (SR) items include a stimulus and stem followed by three to five options from which a student is directed to choose only one or best answer. By redesigning some SR items, it is often possible to both increase the complexity of the item and yield more useful information regarding the level of understanding about the mathematics that a student's response demonstrates. For example, consider the following SR item in which one of the four options is the correct response (Figure 1).

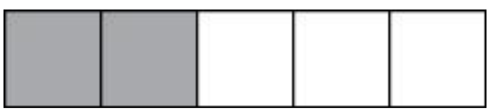
Figure 1.

Which model below best represents the fraction $\frac{2}{5}$?

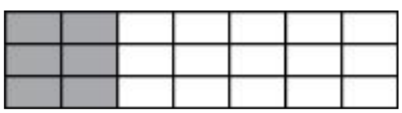
A.



B.



C.

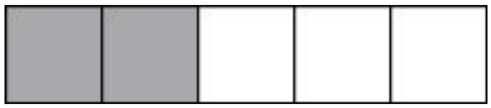


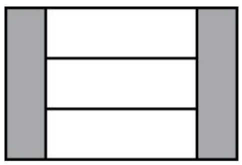
D.

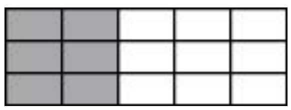
Even if a student does not truly have a deep understanding of what 2/5 means, he or she is likely to choose option B over the rest of the options because it looks to be a more traditional way of representing fractions. By a simple restructuring of this problem into a multi-part item, including a modification to option C, a clearer sense of how deeply a student understands the concept of 2/5 can be ascertained (see Figure 2).

Figure 2.

For numbers 1a-1d, state whether or not each figure has $\frac{2}{5}$ of its whole shaded.

1a.  Ⓨ Yes Ⓝ No

1b.  Ⓨ Yes Ⓝ No

1c.  Ⓨ Yes Ⓝ No

1d.  Ⓨ Yes Ⓝ No

This item is more complex in that a student now has to look at each part separately and decide whether 2/5 can take different forms. By assigning two points to this problem, we can also provide feedback at the item level as to the depth of understanding a student has about simple fractions. The total number of ways to respond to this item is 16. "Guessing" the correct combination of responses is much less likely than for a traditional 4-option selected-response item. The correct response for this item will receive 2 points, and the points will be earned based on the level of understanding the student has demonstrated. A suggested scoring rubric for this item follows in Figure 3.

Figure 3.

Scoring Rubric

Responses to this item will receive 0-2 points, based upon the following:

2 points: YNYN The student has a solid understanding of 2/5 as well as an equivalent form of 2/5.

1 point: YNNN, YYNN, YYYN The student has only a basic understanding of 2/5. Either the student doesn't recognize an equivalent fraction for 2/5 or doesn't understand that all 5 parts must be equal-sized in figure 1b.

0 points: YYYY, YNNY, NNNN, NNYY, NYYN, NYNN, NYYY, NYNN, NNNN, NYNY, NNYN, NNNY The student demonstrates inconsistent understanding of 2/5 or answers "Y" to figure 1d, clearly showing a misunderstanding of what 2/5 means. Figure 1d is considered a "disqualifier" and an answer of "Y" to this part of the item would cancel out any other correct responses as "guesses" on the part of the student.

Other ways to format SR items include providing options in which more than one choice is correct (e.g., 3 out of 8 options), multiple-part yes/no questions, and even multiple-part items that require students to match descriptions of a term or activity to the corresponding option.

It is important to note that a writer should only consider some of these alternate SR formats when, by doing so, they serve to provide more information about a student's knowledge, skills, and ability (KSA) than a simpler 4- or 5-option SR item would demonstrate.

General Guidelines for Developing SR Items

- Each item should be written to focus primarily on one assessment target. Secondary targets are acceptable and should be listed in the item forms as appropriate, but it should be clear to all stakeholders which assessment target is the focal point of the item.
- Items should be appropriate for students in terms of grade-level difficulty, cognitive complexity, and reading level.[1]
- Items should not provide an advantage or disadvantage to a particular group of students. Items should not exhibit or reflect disrespect toward any segment of the population with regard to age, gender, race, ethnicity, language, religion, socioeconomic status, disability, or geographic region.[2]
- Items are expected to include mathematical concepts detailed in the CCSSM of lower grades.
- At grades 3–5, all items should be written so they can be answered without using a calculator
- Items should provide clear and complete instructions to students.
- Each item should be written to clearly elicit the desired evidence of a student's KSA.
- Options should be arranged according to a logical order (e.g., alphabetical, least to greatest value, greatest to least value, length of options).

[1] For mathematics items, the reading level should be approximately one grade level below the grade level of the test, except for specifically assessed mathematical terms or concepts.
[2] This guideline is discussed in greater detail in the Bias/Sensitivity and Accessibility portions of the *Specifications*.

Constructed-Response Items

The main purpose of a constructed-response (CR) item/task is to address targets and claims that are of greater complexity, requiring more analytical thinking and reasoning than an SR item can typically elicit. Additionally, fill-in-the-blank type CR items (CRs) can markedly increase the discrimination factor and reliability of comparable SR items (SRs) by virtually eliminating the "guessing" element of those items.

The *Smarter Balanced* mathematics assessments will include a wide variety of CRs. A primary distinction that will be made in the development of CR items/tasks will be related to the portion of the assessment to which the item/task is assigned. As noted earlier in this document, the design of the assessment includes both a computer-adaptive component and a performance task component. It is desirable to have CRs that cover a variety of content domains in both components. All CRs must be computer delivered and response captured. While most CRs should be designed for scoring using artificial intelligence (AI), some CRs may require handscoring.

In order to distinguish the CR items/tasks that require handscoring from those that utilize AI scoring, the former will be referred to as extended-response (ER) items/tasks. Therefore, a CR designation in mathematics will refer to constructed-response items that can be scored with AI. Both ERs and CRs will be present in the computer-adaptive and performance task components. The length of time these CRs take to administer should typically vary from 1 to 5 minutes, some may require up to 10 minutes to complete.

General Guidelines for Developing CR and ER Items/Tasks

- Each item/task should be written to assess a primary content domain. Secondary content domains are also possible and should be listed in order of prominence when completing the item form.
- Items/tasks should be appropriate for students in terms of grade-level difficulty, cognitive complexity, and reading level.[3]
- Items/tasks should not provide an advantage or disadvantage to a particular group of students. Items should not exhibit or reflect disrespect toward any segment of the population with regard to age, gender, race, ethnicity, language, religion, socioeconomic status, disability, or geographic region.[4]
- Items/tasks are expected to include mathematical concepts detailed in the CCSS of lower grades.
- At grades 3–5, all items should be written so they can be answered without using a calculator.
- Items/tasks should provide clear and complete instructions to students.
- CR items/tasks should be written to clearly elicit the desired evidence of a student’s KSA.
- For CR items, a complete key and/or scoring rubric must be included with the item along with a justification for the solution, as needed.
- For ER items/tasks,[5] a “Sample Top-Score Response” must be included, accompanied by a scoring rubric that details the rationale for awarding each score point in terms of the evidence demonstrated by a student’s response.

[3] For mathematics items, the reading level should be approximately one grade level below the grade level of the test, except for specifically assessed mathematical terms or concepts.
[4] This guideline is discussed in greater detail in the Bias/Sensitivity and Accessibility portions of the *Specifications*.
[5] Scoring guidelines for ERs, as well as PTs, are discussed more thoroughly in the sections of this document that immediately follow the Specification Tables for Claims 2, 3, and 4.

Technology-Enhanced Items/Tasks

Technology-enhanced (TE) items/tasks are desirable when they can provide evidence for mathematical practices that could not be as reliably obtained from SR and CR items. Additionally, components of certain extended-response (ER) items and performance tasks may employ TE tools as part of the task. An expressed desire on the part of the consortium is that the use of TE items in the assessments will ultimately encourage classroom use of authentic mathematical computing tools (e.g., spreadsheets, interactive geometry software) as part of classroom instruction.

A specific section of the *Test Item and Performance Task Specifications* is devoted entirely to the development of TE items. Determining whether a TE item/task is the best vehicle with which to assess a particular piece of evidence will require a careful analysis of the TE section of these *Specifications.*

Performance Tasks

As with TE items/tasks, an entire section of these *Specifications* contains information related to the development of high-quality performance tasks, and a writer must refer to that section when attempting to write these tasks. In short, performance tasks should:

- Integrate knowledge and skills across multiple claims and targets.
- Measure capacities such as depth of understanding, research skills, and/or complex analysis with relevant evidence.
- Require student-initiated planning, management of information/data and ideas, and/or interaction with other materials.
- Reflect a real-world task and/or scenario-based problem.
- Allow for multiple approaches.
- Represent content that is relevant and meaningful to students.
- Allow for demonstration of important knowledge and skills, including those that address 21st century skills, such as critically analyzing and synthesizing information presented in a variety of formats, media, etc.
- Require scoring[6] that focuses on the essence of the Claim(s) and Targets for which the task was written.
- Be feasible for the school/classroom environment.

[6] Scoring rules are described in detail in the Performance Task section of the *Specifications*.

PTs may require up to 135 minutes to administer. This administration time includes a 45 or 90 minute classroom portion and a 45 minute computer-based portion.

The table below represents the general structure of most mathematics PTs.

<table>
<tr><th colspan="3">Mathematics Performance Task</th></tr>
<tr><th>Stimulus</th><th>Information Processing</th><th>Product/Performance</th></tr>
<tr>
<td>graphs
video clips
maps
photos
research reports
geometric figures
2-D and 3-D models
spreadsheets
data bases
areas of math content (alg, geom.)
etc.</td>
<td>Tools
calculators
measurement devices
data analysis software
geometric simulation and construction tools
context/scenario specific simulations
equation editor tool
spreadsheets
etc.
Tasks
comprehension questions
small group discussion/notes
investigation/search (group or indiv.)
analyses
mathematical proofs
etc.</td>
<td>Classroom-Based
essay/report on problem solution w/mathematical justification
oral presentation w/wo graphics, other media
math-based design
graphic displays
2-D, 3-D models
mathematical proof
spreadsheets etc.
Computer-Based
conceptually and/or contextually linked to classroom-based portion of PT
set of 12 – 20 items associated with each PT
written to gather evidence for Claims 2, 3, and 4 represent a range of difficulty levels</td>
</tr>
</table>

Computational Complexity of Mathematics Items and Tasks

In general, items/tasks developed to assess student understanding of core concepts and procedures in mathematics will draw upon grade-level standards to ensure student mastery of this content. Grade-level standards are implicitly included in the assessment targets of Claim 1 for grades 3–5. The assessment targets for Claim 1 correspond directly to the cluster headings contained in the CCSSM, and the technical demand of these Claim 1 items can be consistent with the grade level being assessed. However, when writing more complex tasks (such as those associated with Claims 3 and 4), the computational demand should be lowered and should typically be met by content that was first taught in earlier grades.

Use of Formulas

For some items/tasks, students will be expected to know the formulas that are used to solve problems (e.g., formulas for area, perimeter, volume, etc.). A reference sheet of formulas will not be available to students for these items. However, for more complex problems, a search function may be available (on an item-by-item basis) that allows a student to call up a table of formulas. This table will not include names of the formulas or pictures of the figures for which the formulas are relevant, and will serve only to prompt a student's memory of the correct formula. For some TE items, it is possible that drop-down menus will be available for students to choose a relevant formula for a particular item.

Claim 1: Concepts and Procedures

Claim 1 **— Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.**

Rationale for Claim 1

This claim addresses procedural skills and the conceptual understanding on which developing skills depends. It is important to assess how aware students are of how concepts link together and why mathematical procedures work the way they do. Central to understanding this claim is making the connection to these elements of the mathematical practices as stated in the CCSSM:

Use appropriate tools strategically.

- Use technological tools to explore and deepen their understanding of concepts.

Attend to precision.

- State the meaning of the symbols they choose, including using the equal sign consistently and appropriately.
- Specify units of measure and label axes to clarify the correspondence with quantities in a problem.
- Calculate accurately and efficiently, and express numerical answers with a degree of precision appropriate for the problem context.
  - In the elementary grades, students give carefully formulated explanations to each other.

Look for and make use of structure.

- Look closely to discern a pattern or structure.
  - Young students might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have.
  - Later, students will see 7 x 8 equals the well remembered 7 x 5 + 7 x 3, in preparation for the distributive property.
- See complicated things, such as some algebraic expressions, as single objects or composed of several objects.

Look for and express regularity in repeated reasoning.

- Notice if calculations are repeated.
- Look for both general methods and shortcuts.
  - Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again and conclude they have a repeating decimal.
- Maintain oversight of the process of solving a problem while attending to the details.
- Continually evaluate the reasonableness of intermediate results.

Essential Properties of Items/Tasks that Assess Claim 1

Items/tasks assessing this claim will include SR items, TE items/tasks, and short CR items/tasks that focus on a particular skill or concept. They will also include items that require students to translate between representations of concepts (words, diagrams, symbols) and items that require the identification of structure.

It is important to note that Claim 1 specification tables are the only ones in which a direct connection to the content domains and clusters of the grade-level CCSSM is made. Items/tasks designed to elicit the evidence sought in Claims 2, 3, and 4 will necessarily rely on the content explicated in the Claim 1 specification tables.

Specification tables have been developed for each assessment target associated with Claim 1. These tables are intended to provide guidance to item writers for the development of items/tasks that primarily assess Claim 1. Figure 4 provides the model used for all Claim 1 tables along with an explanation of the metadata that populates each cell of the table.

This is the Claim 1 statement, taken directly from the Content Specifications.

Figure 4.

Claim 1: Concepts and Procedures Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.	
Content Domain: *This cell contains the content domain associated with the specified target. For grades 3–5, eligible domains are: Operations & Algebraic Thinking, Number & Operations – Base Ten, Number & Operations – Fractions, Measurement & Data, and Geometry.*	
Target __ [ ]: *Assessment Target letter and [emphasis], as defined by the Content Specifications. The emphasis designation is identified by "m" for major, "s" for supporting, and "a" for additional. The complete text of the target will populate the rest of this cell.*	
Standards:	*Standards from the CCSSM related to the specified target.*
DOK Target(s):	*Depth-of-Knowledge level(s) assigned to the specified target.*
Evidence Required:	*Statements that define the knowledge, skills, or abilities a student must demonstrate in order to provide evidence in support of one or more aspects of the target and claim.*
Allowable Item Types*:	*The item types allowed for this target (SR, CR, or TE).*
Task Models:	*A task model describes key characteristics or features that items are to have in order to establish a context or problem that elicits the desired evidence from the student. In effect, a task model describes what the prompt is intended to ask of the student, the content or materials (stimuli) that the student is supposed to work with when applying the targeted knowledge, skill, or ability, and any unique interactions that the item must support in order to allow the student to produce the desired response information.* *For every enumerated statement of "evidence required," a corresponding task model will follow. If more than one type of item/task is appropriate for the same evidence statement, then the same number will be assigned. The variables will be the item type and DOK level associated with the task model.*
Allowable Stimulus Materials:	*This cell lists the kinds of stimuli that can be used. It is not to be considered a complete list, but suggests various types.*
Allowable Disciplinary Vocabulary:	*This cell suggests mathematics-specific vocabulary that students are expected know, as related to the target.*
Allowable Tools:	*This cell identifies allowable tools that students may use in working with the item/task (e.g., ruler, protractor, etc.).*
Target-Specific Attributes:	*This cell identifies specific attributes, related to the target, which could include limitations on the content or other considerations.*
Key Nontargeted Constructs:	*This cell identifies knowledge and skills the student needs in order to respond, but which are not scored for the specified target.*
Accessibility Concerns:	*This cell identifies possible concerns for students with disabilities or those with other accessibility issues.*
Sample Items:	*This cell contains item codes that are "hot-button" links with access to sample items for the specified target.*

The Standard numbers, DOK numbers, and Item Types are all intended to be "hot-button" links to the complete text of each component.

*SR = selected-response item; CR = constructed-response item; TE = technology-enhanced item; ER = extended-response item; PT = performance task

How to Complete the Item Form for Claim 1 Targets

Items that are written to Claim 1 assessment targets must follow the guidelines contained throughout these specifications. Additionally, item writers must complete an Item Form for every submitted item. Figure 5 provides the model used for all Claim 1 item forms along with an explanation of the metadata that populates each cell of the form.

Figure 5.

Sample Item ID:	MAT.GR.IT.1.CDOMA.T.xxx (see below)
Grade:	Specify the 2-digit grade level (HS for high school).
Claim(s):	Enter the number and text of the primary *Smarter Balanced* claim. If more than one claim is part of the item/task, the first number must represent the **primary claim**, with secondary and tertiary claims listed by order of importance.
Assessment Target(s):	Enter the *Smarter Balanced* target alpha character(s) and the text of the *Smarter Balanced* target(s).
Content Domain:	Enter the primary CCSSM domain associated with the claim and target.
Standard(s):	Enter the number(s) of the CCSSM standard(s).
Mathematical Practice(s):	Specify the mathematical practices (1–8) associated with the item/task.
DOK:	Specify the Depth of Knowledge level (1–4) of the item/task.
Item Type:	Specify the item type (SR, CR, TE).
Score Points:	Specify the total point value of the item.
Difficulty:	Specify the estimated difficulty of item (L=Low, M=Medium, H=Hard). See below for further explanation of coding.
Key:	Specify the correct key for SR items or indicate "See Sample Top-Score Response" for multi-point items/tasks.
Stimulus/Source:	Specify any stimulus material used and/or source required for factual information. All sources must be reliable and reproducible. If none, leave blank.
Target-Specific Attributes (e.g., accessibility issues):	Specify any target-specific attributes (e.g., accessibility issues).
Notes:	Add any notes here that you believe will aid in understanding the purpose of this sample item. For TE items, include the TE template name here.

These are all intended to be "hot-button" links to the complete text of each component.

Sample Item ID: Specify the sample item ID "MAT.GR.IT.C.CDOMA.T.xxx"

MAT – Mathematics
GR – 03, 04, 05, 06, 07, 08, or HS
IT – Item type (SR, CR, TE, ER, or PT)
C – Claim number 1, 2, 3, or 4
CDOMA – Content Domain letters from CCSSM (e.g., OA, MD, …must be five places, lead with zeros until all five places are filled)

Note: For PTs, the content domain field is filled with the task name abbreviation.

T – Primary Assessment target alpha (A, B, C, D, etc.)
xxx – Item number. Leave alone for now; will be assigned after acceptance.

Difficulty – Base the level on the percent of students that would be expected to get the item/task correct or would earn the maximum number of points as follows:
Low – greater than 70% Medium – 40% to 70% Hard – less than 40%

Item/Task: Use this space for the stem, stimulus, and/or options. The font size for items/tasks is Verdana, 14-point.

Key and Distractor Analysis or Scoring Rubric for Multi-Part Items/Tasks: The information in this section will vary, depending on the item type below. For traditional 1-point SR items: Include the key in both the item form and in this section and provide complete rationales for each distractor. For multi-point SR items: Include a rubric and justification for each score point. Refer to the details presented in Figure 3. For short CR items/tasks: Provide a sample top-score response, followed by a rubric and justification for each score point (see directions after Claim 2 Item Form). For TE items/tasks: Provide a sample top-score response and description of the interaction(s) the student must demonstrate to score each point (may be only 1 point).

Claim 2: Problem Solving

Claim 2 — Students can solve a range of complex, well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies.

Claim 2 Overview

Assessment items and tasks focused on Claim 2 include problems in pure mathematics and problems set in context. *Problems* are presented as items and tasks that are well-posed (that is, problem formulation is not necessary) and for which a solution path is not immediately obvious.[7] These problems require students to construct their own solution pathway rather than follow a provided one. Such problems will therefore be unstructured, and students will need to select appropriate conceptual and physical tools to use.

Essential Properties of Items/Tasks that Assess Claim 2

Claim 2 will be assessed using a combination of SR items, TE items, CR items/tasks, and ER items/tasks that focus on making sense of problems and using perseverance in solving them.

To preserve the focus and coherence of the standards as a whole, Claim 2 items/tasks must draw clearly on knowledge and skills that are articulated in the *Smarter Balanced* content standards. At each grade level, the content standards offer natural and productive settings for generating evidence for Claim 2. Items/tasks generating evidence for Claim 2 in a given grade may also draw upon knowledge and skills articulated in the progression of standards up to that grade.

The intent is that each of the targets should not lead to a separate item/task, but will provide evidence for **several** of the assessment targets defined for Claim 2. It is in *using* content from different areas, including work studied in earlier grades, that students demonstrate their problem-solving proficiency. For this reason, the specification tables will look somewhat different from the Claim 1 tables. Specifically, a separate table is not relevant at the target level, so all targets are included in a single, grade-level table for Claim 2. Another important distinction between Claim 1 specification tables (which link the content to other claims) is that the evidence required of students to satisfy Claim 2 centers around specific statements of the ***mathematical practices*** contained in the CCSSM. These statements are found in the cell labeled "Rationale," again relying on the Content Specifications for clarity.

As stated above, Claim 1 specification tables are the only ones in which a direct connection to the content domains and clusters of the grade-level CCSSM is made. Therefore, items/tasks designed to elicit the evidence sought in Claim 2 will rely on the content explicated in the Claim 1 specification tables, including content specified in earlier grades.

Figure 6 provides the model used for all Claim 2 tables. Most of the information contained in Figure 6 will be repeated in all Claim 2 tables for grades 3–5. Notes have been added to specific cells in order to clarify the information and/or source of the metadata contained in those cells.

[7] Schoenfeld, A. H. (1985). *Mathematical problem solving.* Orlando, FL: Academic Press.

Figure 6.

Primary Claim 2: Problem Solving Students can solve a range of well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies.
Secondary Claim(s): Items/tasks written primarily to assess Claim 2 will necessarily involve some Claim 1 content targets. Related Claim 1 targets should be listed below the Claim 2 targets in the item form. If Claim 3 or 4 targets are also directly related to the item/task, list those following the Claim 1 targets in order of prominence.
Primary Content Domain: Each item/task should be classified as having a primary, or dominant, content focus. The content should draw upon the knowledge and skills articulated in the progression of standards leading up to Grade [3, 4, 5].
Secondary Content Domain(s): While tasks developed to assess Claim 2 will have a primary content focus, components of these tasks will likely produce enough evidence for other content domains that a separate listing of these content domains needs to be included where appropriate.
Assessment Targets: Any given item/task should provide evidence for several Claim 2 assessment targets. Each of the following targets should not lead to a separate task: it is in *using* content from different areas, including work studied in earlier grades, that students demonstrate their problem solving proficiency. Multiple targets should be listed in order of prominence as related to the item/task.
Target A: Apply mathematics to solve well-posed problems arising in everyday life, society, and the workplace. (DOK 2, 3) Under Claim 2, the problems should be completely formulated, and students should be asked to find a solution path from among their readily available tools. **Target B: Select and use appropriate tools strategically. (DOK 1, 2)** Tasks used to assess this target should allow students to find and choose tools; for example, using a "Search" feature to call up a formula (as opposed to including the formula in the item stem) or using a protractor in physical space. **Target C: Interpret results in the context of a situation. (DOK 2)** Tasks used to assess this target should ask students to link their answer(s) back to the problem's context. In early grades, this might include a judgment by the student of whether to express an answer to a division problem using a remainder or not based on the problem's context. In later grades, this might include a rationalization for the domain of a function being limited to positive integers based on a problem's context (e.g., understanding that the number of buses required for a given situation cannot be 32½, or that the negative values for the independent variable in a quadratic function modeling a basketball shot have no meaning in this context). **Target D: Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flowcharts, or formulas). (DOK 1, 2, 3)** For Claim 2 tasks, this may be a separate target of assessment explicitly asking students to use one or more potential mappings to understand the relationship between quantities. In some cases, item stems might suggest ways of mapping relationships to scaffold a problem for Claim 2 evidence.

Relevant Verbs:	understand (often in conjunction with one or more other relevant verbs), solve, apply, describe, illustrate, interpret, and analyze

	[Note: This list of verbs came directly from the Content Specifications, immediately preceding the list of targets for Claim 2, "Relevant Verbs for Identifying Content Clusters and/or Standards for Claim 2."]
DOK Target(s):	1, 2, 3 *[Note: These are hot-button links to the full text.]*
Claim 2 Rationale:	**Mathematical Practice 1: Make sense of problems and persevere in solving them.** Mathematically proficient students: • explain to themselves the meaning of a problem and look for entry points to its solution. • analyze givens, constraints, relationships, and goals. • make conjectures about the form and meaning of the solution attempt. • plan a solution pathway rather than simply jumping into a solution. • consider analogous problems and try special cases and simpler forms of insight into the solutions. • monitor and evaluate their progress and change course if necessary. • transform algebraic expressions or change the viewing window on their graphing calculator to get information. • explain correspondences between equations, verbal descriptions, tables, and graphs. • draw diagrams of important features and relationships, graph data, and search for regularity or trends. • use concrete objects or pictures to help conceptualize and solve a problem. • check their answers to problems using a different method. • ask themselves, "Does this make sense?" • understand the approaches of others in solving complex problems and identify correspondences between approaches. **Mathematical Practice 5: Use appropriate tools strategically.** Mathematically proficient students: • consider available tools when solving a mathematical problem. (Tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software.) • are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. • detect possible errors by using estimations and other mathematical knowledge.

	Mathematical Practice 7: Look for and make use of structure.
	Mathematically proficient students:
	• look closely to discern a pattern or structure.
	    ○ Young students might notice that three and seven more is the same amount as seven and three more or they may sort a collection of shapes according to how many sides the shapes have.
	    ○ Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for the distributive property.
	    ○ In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems.
	• step back for an overview and can shift perspective.
	• see complicated things, such as some algebraic expressions, as single objects or composed of several objects.
	
	Mathematical Practice 8: Look for and express regularity in repeated reasoning.
	Mathematically proficient students:
	• notice if calculations are repeated.
	• look for both general methods and shortcuts.
	    ○ Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations and conclude they have a repeated decimal.
	    ○ Middle school students might abstract the equation (y-2)/(x-1)=3 by paying attention to the calculation of slope as they repeatedly check whether the points are on the line through (1, 2) with a slope 3.
	• maintain oversight of the process of solving a problem, while attending to the details.
	• continually evaluate the reasonableness of intermediate results.

Allowable Item Types*:	SR, CR, ER, TE *[Note: These are hot-button links to the full text.]*
Task Models:	**Problems in pure mathematics.** These are well-posed problems within mathematics where the student must find an approach, choose which mathematical tools to use, carry the solution through, and explain the results. **Design problems.** These are well-posed problems within a real- or fantasy-world context where the student must find an approach, choose which mathematical tools to use, carry the solution through, and explain the results.

	Planning problems. Planning problems involve the coordinated analysis of time, space, cost, and people. They are design tasks with a time dimension added. Well-posed problems of this kind assess the student's ability to make the connections needed between different parts of mathematics. Note: This is not a complete list; other types of tasks that fit the criteria above may be included. *[Writers/developers should look for other kinds of evidence that support the Claim 2 targets.]*
Allowable Tools:	protractor, ruler, spreadsheet, mathematical software
Key Nontargeted Constructs:	*[Note: This cell identifies knowledge and skills the student needs in order to respond, but which are not scored for specified target(s).]*
Claim-Specific Attributes:	Items/tasks must be real-world or scenario-based (e.g., fantasy) and should take 5–15 minutes to solve.
Accessibility Concerns:	Real-world problems may sometimes be text heavy. Translation tools and dictionaries should be available to ELL students. Text readers should be available to students, as necessary.
Sample Items:	*[Note: Item codes will be hot-button links to sample items that illustrate Claim 2.]*

*SR = selected-response item; CR = constructed-response item; TE = technology-enhanced item; ER = extended-response item; PT = performance task

How to Complete the Item Form for Claim 2

Items/tasks that are written to Claim 2 assessment targets must follow the guidelines contained throughout these specifications. Additionally, item writers must complete an Item Form for every submitted item/task. Figure 7 provides the model used for all Claim 2 items/tasks, along with an explanation of the metadata that populates each cell of the form.

Figure 7.

Sample Item ID:	MAT.GR.IT.2.CDOMA.T.xxx (see below)
Grade:	Specify the 2-digit grade level (HS for high school).
Primary Claim:	**Claim 2: Problem Solving** Students can solve a range of well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies.
Secondary Claim(s):	List Claim 1 targets first, then Claim 3 or 4 targets (as needed), in order of prominence. May be left blank.
Primary Content Domain:	List the primary content domain of the item/task, as specified in the CCSSM.
Secondary Content Domain(s):	List additional content domains of the given item/task, as needed. May be left blank.
Assessment Target(s):	Multiple targets should be listed in order of prominence as related to the item/task. List the claim number first, then the target associated with that claim, accompanied

	by the text of the target (e.g., 2 C: Interpret results in the context of a situation).
Standard(s):	Enter the number(s) of the CCSSM standard(s).
Mathematical Practice(s):	Specify the mathematical practices (1–8) associated with the item/task.
DOK:	Specify the Depth of Knowledge level (1–4) of the item/task.
Item Type:	Specify the item type (SR, CR, ER, TE).
Score Points:	Specify the total point value of the item.
Difficulty:	Specify the estimated difficulty of item (L=Low, M=Medium, H=Hard). See below for further explanation of coding.
Key:	Specify the correct key for SR items or indicate "See Sample Top-Score Response" for multi-point items/tasks.
Stimulus/Source:	Specify any stimulus material used and/or source required for factual information. All sources must be reliable and reproducible. If none, leave blank.
Target-Specific Attributes (e.g., accessibility issues):	Specify any target-specific attributes (e.g., accessibility issues).
Notes:	Add any notes here that you believe will aid in understanding the purpose of this sample item. For TE items, include the TE template name here.

These are all intended to be "hot-button" links to the complete text of each component.

Sample Item ID: Specify the sample item ID "MAT.GR.IT.C.CDOMA.T.xxx"

MAT – Mathematics
GR – 03, 04, 05, 06, 07, 08, or HS
IT – Item type (SR, CR, TE, ER, or PT)
C – Claim number 1, 2, 3, or 4
CDOMA – Content Domain letters from CCSS (e.g., OA, MD, …must be five places, lead with zeros until all five places are filled)

Note: For PTs, the content domain field is filled with the task name abbreviation.

T –Assessment target alpha (A, B, C, D, etc.)
xxx – Item number. Leave alone for now; will be assigned after acceptance.

Difficulty – Base the level on the percent of students that would be expected to get the item/task correct or would earn the maximum number of points as follows:
Low – greater than 70% Medium – 40% to 70% Hard – less than 40%

Item/Task: Use this space for the stem, stimulus, and/or options. The font size for items/tasks is Verdana, 14-point.

Key and Distractor Analysis or Scoring Rubric for Multi-Part Items/Tasks:

The information in this section will vary, dependent on the item type.

For traditional 1-point SR items: Include the key in both the item form and in this section, and provide complete rationales for each distractor.

For multi-point SR items: Include a rubric and justification for each score point. Refer to the details presented in Figure 3.

For CR and ER items/tasks: Provide a sample top-score response, followed by a rubric and justification for each score point.

For TE items/tasks: Provide a sample top-score response and description of the interaction(s) the student must demonstrate to score each point (may be only 1 point).

Sample Top-Score Response:
Provide an example of a complete and thorough top-score response. The language of this response does not need to be "kid-speak," but it should model what is expected from a student at the specified grade.

Scoring Rubric:
The language of the rubric should:

- focus on the essence of the primary claim;
- address the requirements of the specific target(s);
- distinguish between different levels of understanding and/or performance; and
- contain relevant information/details/numbers that support different levels of competency related to the item/task.

Claim 3: Communicating Reason

Claim 3 — Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.

Claim 3 Overview

Claim 3 refers to a recurring theme in the CCSSM content and practice standards—the ability to construct and present a clear, logical, convincing argument. For older students, this may take the form of a rigorous, deductive proof based on clearly stated axioms. For younger students, this will involve more informal justifications. Assessment tasks that address this claim will typically present a claim and ask students to provide, for example, a justification or counterexample.

Essential Properties of Tasks that Assess Claim 3

> Rigor is about precision in argument: first avoiding making false statements, then saying more precisely what one assumes, and providing the sequence of deductions one makes on this basis. Assessments should also include tasks that examine a student's ability to analyze a provided explanation, identify flaws, and correct them.[8]

Claim 3 will be assessed using a combination of SR, CR, TE, PT, and ER items/tasks that focus on mathematical reasoning. Some tasks will require students to construct chains of reasoning without specific guidance being provided throughout the task.

Claim 3 items/tasks must draw clearly on knowledge and skills that are articulated in the content standards. At each grade level, the content standards offer natural and productive settings for generating evidence for Claim 3. Items/tasks generating evidence for Claim 3 in a given grade may also draw upon knowledge and skills articulated in the progression of standards up to that grade.

The intent is that each of the targets should not lead to a separate item/task, but provide evidence for **several** of the assessment targets defined for Claim 3. For this reason, a separate table is not relevant at the target level, so all targets are included in a single grade-level table for Claim 3. For this claim (as with Claims 2 and 4), the statements found in the table cell labeled "Rationale" are drawn from the ***mathematical practices*** contained in the CCSSM.

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[8] Content Specifications for the Summative Assessment of the *Common Core State Standards for Mathematics* (Draft), December 9, 2011.

Since Claim 1 specification tables are the only ones in which a direct connection to the content domains and clusters of the grade-level CCSSM is made, items/tasks designed to elicit the evidence sought in Claim 3 will necessarily rely on the content explicated in the Claim 1 specification tables.

Figure 8 provides the model used for all Claim 3 tables. Most of the information contained in Figure 8 will be repeated in all Claim 3 tables for grades 6–8. Notes have been added to specific cells in order to clarify the information and/or source of the metadata contained in those cells.

Figure 8.

Primary Claim 3: Communicating Reasoning Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.
Secondary Claim(s): Tasks written primarily to assess Claim 3 will necessarily involve Claim 1 content targets. Related Claim 1 targets should be listed below the Claim 3 targets in the item form. If Claim 2 or Claim 4 targets are also directly related to the task, list those following the Claim 1 targets in order of prominence.
Primary Content Domain: Each task should be classified as having a primary, or dominant, content focus. The content should draw upon the knowledge and skills articulated in the progression of standards leading up to Grade [3, 4, 5].
Secondary Content Domain(s): While tasks developed to assess Claim 3 will have a primary content focus, components of these tasks will likely produce enough evidence for other content domains that a separate listing of these content domains needs to be included where appropriate.
Assessment Targets: Any given task should provide evidence for several of the following assessment targets; each of the following targets should not lead to a separate task. Multiple targets should be listed in order of prominence as related to the task.
Target A: Test propositions or conjectures with specific examples. (DOK 2) Tasks used to assess this target should ask for specific examples to support or refute a proposition or conjecture (e.g., An item stem might begin, "Provide 3 examples to show why/how…"). **Target B: Construct, autonomously,[9] chains of reasoning that will justify or refute propositions or conjectures.[10] (DOK 3, 4)** Tasks used to assess this target should ask students to develop a chain of reasoning to justify or refute a conjecture. Tasks for Target B might include the types of examples called for in Target A as part of this reasoning, but they should do so with a lesser degree of scaffolding than tasks that assess Target A alone. Some tasks for this target will ask students to formulate and justify a conjecture. **Target C: State logical assumptions being used. (DOK 2, 3)** Tasks used to assess this target should ask students to use stated assumptions, definitions,

[9] By "autonomous" we mean that the student responds to a single prompt, without further guidance within the task.
[10] At the secondary level, these chains may take a successful student 10 minutes to construct and explain. Times will be somewhat shorter for younger students, but still giving them time to think and explain. For a minority of these tasks, subtasks may be constructed to facilitate entry and assess student progress towards expertise. Even for such "apprentice tasks," part of the task will involve a chain of autonomous reasoning that takes at least 5 minutes.

and previously established results in developing their reasoning. In some cases, the task may require students to provide missing information by researching or providing a reasoned estimate. **Target D: Use the technique of breaking an argument into cases. (DOK 2, 3)** Tasks used to assess this target should ask students to determine under what conditions an argument is true, to determine under what conditions an argument is not true, or both. **Target E: Distinguish correct logic or reasoning from that which is flawed and—if there is a flaw in the argument—explain what it is. (DOK 2, 3, 4)** Tasks used to assess this target present students with one or more flawed arguments and ask students to choose which (if any) is correct, explain the flaws in reasoning, and/or correct flawed reasoning. **Target F: Base arguments on concrete referents such as objects, drawings, diagrams, and actions. (DOK 2, 3)** In earlier grades, the desired student response might be in the form of concrete referents. In later grades, concrete referents will often support generalizations as part of the justification rather than constituting the entire expected response. **Target G: At later grades, determine conditions under which an argument does and does not apply. (For example, area increases with perimeter for squares, but not for all plane figures.) (DOK 3, 4)** Tasks used to assess this target will ask students to determine whether a proposition or conjecture always applies, sometimes applies, or never applies and provide justification to support their conclusions. Targets A and B will likely be included also in tasks that collect evidence for Target G.	
Relevant Verbs:	Understand, explain, justify, prove, derive, assess, illustrate, and analyze *[Note: This list of verbs came directly from the Content Specifications, immediately preceding the list of targets for Claim 3, "Relevant Verbs for Identifying Content Clusters and/or Standards for Claim 3."]*
DOK Target(s):	2, 3, 4 *[Note: These are hot-button links to the full text.]*
Claim 3 Rationale:	**Mathematical Practice 3: Construct viable arguments and critique the reasoning of others.** Mathematically proficient students: • understand and use stated assumptions, definitions, and previously established results in constructing arguments. • make conjectures and build a logical progression of statements to explore the truth of their conjectures. • analyze situations by breaking them into cases. • recognize and use counterexamples. • justify their conclusions, communicate them to others, and respond to the arguments of others. • reason inductively about data, making plausible arguments that take into account the context from which the data arose. • compare the effectiveness of plausible arguments. • distinguish correct logic or reasoning from that which is flawed and, if there is a flaw, explain what it is.

	o Elementary students construct arguments using concrete referents such as objects, drawings, diagrams, and actions. o Later students learn to determine domains to which an argument applies. • listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve arguments. **Mathematical Practice 6: Attend to precision.** Mathematically proficient students: • communicate precisely to others. • use clear definitions in discussion with others and in their own reasoning. • state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. • specify units of measure and label axes to clarify the correspondence with quantities in a problem. • calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. o In the elementary grades, students give carefully formulated explanations to each other. o In high school, students have learned to examine claims and make explicit use of definitions.
Allowable Item Types*:	SR, CR, ER, TE, PT *[Note: These are hot-button links to the full text.]*
Task Models:	**Proof and justification tasks:** These begin with a proposition, and the task is to provide a reasoned argument why the proposition is or is not true. In some tasks, students may be asked to characterize the domain for which the proposition is true. **Critiquing tasks:** Some flawed reasoning is presented, and the task is to correct and improve it. **Mathematical investigations:** Students are presented with a phenomenon and are invited to formulate conjectures about it. They are then asked to prove one of their conjectures. Note: This is not a complete list; other types of tasks that fit the criteria above may be included. *[Writers/developers should look for other kinds of evidence that support the Claim 3 targets.]*
Allowable Tools:	protractor, ruler, spreadsheet, mathematical software
Key Nontargeted Constructs:	*[Note: This cell identifies knowledge and skills the student needs in order to respond, but which are not scored for specified target(s).]*
Claim-Specific Attributes:	Tasks should be designed to take 10–20 minutes to solve. The computational demand on these tasks should focus on the skill level typically expected for Claim 1 tasks for grades up to the specified

	grade, yet be consistent with the content domain emphases of that grade.
Accessibility Concerns:	Problems that require students to communicate reasoning may sometimes be text heavy. Translation tools and dictionaries should be available to ELL students. Text readers should be available to students as necessary.
Sample Items:	*[Note: Item codes will be hot-button links to sample items that illustrate Claim 3.]*

*SR = selected-response item; CR = constructed-response item; TE = technology-enhanced item; ER = extended-response item; PT = performance task

How to Complete the Item Form for Claim 3

Items/tasks that are written to Claim 3 assessment targets must follow the guidelines contained throughout these specifications. Additionally, item writers must complete an Item Form for every submitted item/task. Figure 9 provides the model used for all Claim 3 items/tasks along with an explanation of the metadata that populates each cell of the form.

Figure 9.

Sample Item ID:	MAT.GR.IT.3.CDOMA.T.xxx
Grade:	Specify the 2-digit grade level (HS for high school).
Primary Claim:	**Claim 3: Communicating Reasoning** Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.
Secondary Claim(s):	List Claim 1 targets first, then Claim 2 or 4 targets (as needed), in order of prominence. May be left blank.
Primary Content Domain:	List the primary content domain of the task, as specified in the CCSSM.
Secondary Content Domain(s):	List additional content domains of the given task, as needed. May be left blank.
Assessment Target(s):	Multiple targets should be listed in order of prominence as related to the item/task. List the claim number first, then the target associated with that claim, accompanied by the text of the target (e.g., 3 C: State logical assumptions being used.)
Standard(s):	Enter the number(s) of the CCSSM standard(s).
Mathematical Practice(s):	Specify the mathematical practices (1–8) associated with the item/task.
DOK:	Specify the Depth of Knowledge level (1–4) of the item/task.
Item Type:	Specify the item type (SR, CR, ER, TE, PT).
Score Points:	Specify the total point value of the item.
Difficulty:	Specify the estimated difficulty of item (L=Low, M=Medium, H=Hard). See below for further explanation of coding.
Key:	Specify "See Sample Top-Score Response" for Claim 3 tasks.
Stimulus/Source:	Specify any stimulus material used and/or source required for factual information. All sources must be reliable and reproducible. If none, leave blank.
Claim-Specific Attributes (e.g., accessibility issues):	Specify any target-specific attributes (e.g., accessibility issues).
Notes:	Add any notes here that you believe will aid in understanding the purpose of this sample item. For TE items, include the TE template name here.

These are all intended to be "hot-button" links to the complete text of each component.

Sample Item ID: Specify the sample item ID "MAT.GR.IT.C.CDOMA.T.xxx"

MAT – Mathematics
GR – 03, 04, 05, 06, 07, 08, or HS
IT – Item type (SR, CR, TE, ER, or PT)
C – Claim number 1, 2, 3, or 4
CDOMA – Content Domain letters from CCSS (e.g., OA, MD, …must be five places, lead with zeros until all five places are filled)

Note: For PTs, the content domain field is filled with the task name abbreviation.

T – Assessment target alpha (A, B, C, D, etc.)
xxx – Item number. Leave alone for now; will be assigned after acceptance.

Difficulty – Base the level on the percent of students that would be expected to get the item/task correct or to earn the maximum number of points as follows:
Low – greater than 70%   Medium – 40% to 70%   Hard – less than 40%

Item/Task: Use this space for the stem, stimulus, and/or options. The font size for items/tasks is Verdana, 14-point.

Sample Top-Score Response: Provide an example of a complete and thorough top-score response. The language of this response does not need to be "kid-speak," but it should model what is expected from a student at the specified grade.

Scoring Rubric:

The language of the rubric should:

- focus on the essence of the primary claim;
- address the requirements of the specific target(s);
- distinguish between different levels of understanding and/or performance; and
- contain relevant information/details/numbers that support different levels of competency related to the item/task.

Additionally, the scoring rubric should reflect the values set out for Claim 3, giving substantial weight to the quality and precision of the reasoning in several of the following:

- an explanation of the assumptions made;
- the construction of conjectures that appear plausible, where appropriate;
- the quality of the examples that the student constructs in order to evaluate the proposition or conjecture;
- the reasoning that the student uses to describe flaws or gaps in an argument;
- the clarity and precision with which the student constructs a logical sequence of steps to show how the assumptions lead to the acceptance or refutation of a proposition or conjecture;
- the precision with which the student describes the domain of validity of the proposition or conjecture.

Claim 4: Modeling and Data Analysis

Claim 4 — Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.

Claim 4 Overview

Modeling is the bridge across the "school math"/"real world" divide that has been missing from many mathematics curricula and assessments.[11] It is the twin of *mathematical literacy*, the focus of the PISA international comparison tests in mathematics. CCSSM features modeling as both a mathematical practice at all grades and a content focus in high school.

Essential Properties of Tasks that Assess Claim 4

In the real world, problems do not come neatly "packaged." Real-world problems are complex and often contain insufficient or superfluous data. Tasks designed primarily to assess Claim 4 will involve *formulating* a problem that is tractable using mathematics; that is, formulating a model. This will usually involve making assumptions and simplifications. Students will need to select from the data at hand or estimate data that are missing. (Such tasks are therefore distinct from the well-formulated problem-solving tasks described in Claim 2.) Students will identify variables in a situation and construct relationships between them. Once students have formulated the problem, they will tackle it (often in a decontextualized manner) before interpreting their results and then checking the results for reasonableness.

Claim 4 tasks will often involve more than one content domain and will draw upon knowledge and skills articulated in the progression of standards up to that grade, with strong emphasis on the major work of previous grades.

Claim 4 will be assessed both by performance tasks (each lasting up to 135 minutes) and by constructed response and extended response items that focus on modeling and data analysis. CR and ER tasks should be designed so that a successful student will complete each one in 5–15 minutes.

The intent is that each of the Claim 4 targets should not lead to a separate task, but provide evidence for **several** of the assessment targets defined for Claim 4. For this reason, a separate table is not relevant at the target level, so all targets are included in a single grade-level table for Claim 4. For this claim (as with Claims 2 and 3), the statements found in the table cell labeled "Rationale" are drawn from the ***mathematical practices*** contained in the CCSSM.

Since Claim 1 specification tables are the only ones in which a direct connection to the content domains and clusters of the grade-level CCSSM is made, tasks designed to elicit the evidence sought in Claim 4 will necessarily rely on the content explicated in the Claim 1 specification tables.

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[11] In their everyday life and work, most adults use none of the mathematics they are first taught after age 11. They often do not see the mathematics that they do use (in planning, personal accounting, design, thinking about political issues, etc.) as mathematics.

Figure 10 provides the model used for all Claim 4 tables. Most of the information contained in Figure 10 will be repeated in all Claim 4 tables for grades 3–5. Notes have been added to specific cells in order to clarify the information and/or source of the metadata contained in those cells.

Figure 10.

Primary Claim 4: Modeling and Data Analysis Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.
Secondary Claim(s): Tasks written primarily to assess Claim 4 will necessarily involve Claim 1 content targets. Related Claim 1 targets should be listed below the Claim 4 targets in the item form. If Claim 2 or Claim 3 targets are also directly related to the task, list those following the Claim 1 targets in order of prominence.
Primary Content Domain: Each task should be classified as having a primary, or dominant, content focus. The content should draw upon the knowledge and skills articulated in the progression of standards leading up to Grade [3, 4, 5], with strong emphasis on the major work of previous grades.
Secondary Content Domain(s): While tasks developed to assess Claim 4 will have a primary content focus, components of these tasks will likely produce enough evidence for other content domains that a separate listing of these content domains needs to be included where appropriate.
Assessment Targets: Any given task should provide evidence for several of the following assessment targets; each of the following targets should not lead to a separate task. Multiple targets should be listed in order of prominence as related to the task.
Target A: Apply mathematics to solve problems arising in everyday life, society, and the workplace. (DOK 2, 3) Problems used to assess this target for Claim 4 should not be completely formulated (as they are for the same target in Claim 2), and require students to extract relevant information from within the problem and find missing information through research or the use of reasoned estimates. **Target B: Construct, autonomously, chains of reasoning to justify mathematical models used, interpretations made, and solutions proposed for a complex problem. (DOK 2, 3, 4)** At the secondary level, these chains should typically take a successful student 10 minutes to complete. Times will be somewhat shorter for younger students, but still give them time to think and explain. *For a minority of these tasks*, subtasks may be constructed to facilitate entry and assess student progress towards expertise. Even for such "apprentice tasks," part of the task will involve a chain of autonomous reasoning that takes at least 5 minutes. **Target C: State logical assumptions being used. (DOK 1, 2)** Tasks used to assess this target ask students to use stated assumptions, definitions, and previously established results in developing their reasoning. In some cases, the task may require students to provide missing information by researching or providing a reasoned estimate. **Target D: Interpret results in the context of a situation. (DOK 2, 3)** Tasks used to assess this target should ask students to link their answer(s) back to the problem's context. (See Claim 2, Target C, for further explication.) **Target E: Analyze the adequacy of and make improvements to an existing model or**

develop a mathematical model of a real phenomenon. (DOK 3, 4)
Tasks used to assess this target ask students to investigate the efficacy of existing models (e.g., develop a way to analyze the claim that a child's height at age 2 doubled equals his/her adult height) and suggest improvements using their own or provided data.

Other tasks for this target will ask students to develop a model for a particular phenomenon (e.g., analyze the rate of global ice melt over the past several decades and predict what this rate might be in the future).

Longer constructed-response items and extended performance tasks should be used to assess this target.

Target F: Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flowcharts, or formulas). (DOK 1, 2, 3)
Unlike Claim 2, where this target might appear as a separate target of assessment (see Claim 2, Target D), it will be embedded in a larger context for tasks in Claim 4. The mapping of relationships should be part of the problem posing and solving related to Claim 4, Targets A, B, E, and G.

Target G: Identify, analyze and synthesize relevant external resources to pose or solve problems. (DOK 3, 4)
Especially in extended performance tasks (those requiring up to 120 minutes to complete), students should have access to external resources to support their work in posing and solving problems (e.g., finding or constructing a set of data or information to answer a particular question or looking up measurements of a structure to increase precision in an estimate for a scale drawing). Constructed-response items should incorporate "hyperlinked" information to provide additional detail (both relevant and extraneous) for solving problems in Claim 4.

Relevant Verbs:	model, construct, compare, investigate, build, interpret, estimate, analyze, summarize, represent, solve, evaluate, extend, and apply *[Note: This list of verbs came directly from the Content Specifications, immediately preceding the list of targets for Claim 4, "Relevant Verbs for Identifying Content Clusters and/or Standards for Claim 4."]*
DOK Target(s):	1, 2, 3, 4 *[Note: These are hot-button links to the full text.]*
Claim 4 Rationale:	**Mathematical Practice 2: Reason abstractly and quantitatively.** Mathematically proficient students: • make sense of quantities and their relationships in problem situations. • bring two complementary abilities to bear on problems involving quantitative relationships: ○ *Decontextualize* (abstract a given situation and represent it symbolically; and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents) and ○ *Contextualize* (pause as needed during the manipulation process in order to probe into the referents for the symbols involved). • use quantitative reasoning that entails creating a coherent representation of the problem at hand, considering the units involved, and attending to the meaning of quantities (not just

how to compute them).

- know and flexibly use different properties of operations and objects.

Mathematical Practice 4: Model with mathematics.
Mathematically proficient students:

- apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.
    - In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community.
    - By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another.
- make assumptions and approximations to simplify a complicated situation, realizing that these may need revision later.
- identify important quantities in a practical situation
- map relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas.
- analyze those relationships mathematically to draw conclusions.
- interpret their mathematical results in the context of the situation.
- reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Mathematical Practice 5: Use appropriate tools strategically.
Mathematically proficient students:

- consider available tools when solving a mathematical problem. (Tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software.)
- are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations.
- detect possible errors by using estimations and other mathematical knowledge.
- know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data.
- identify relevant mathematical resources and use them to pose or solve problems.
- use technological tools to explore and deepen their understanding of concepts.

Allowable Item Types*:	PT, CR, ER, TE *[Note: These are hot-button links to the full text.]*
Task Models:	**Make decisions from data:** These tasks require students to select from a data source, analyze the data, and draw reasonable conclusions from it. This will often result in an *evaluation* or *recommendation*. The purpose of these tasks is not to provide a setting for the student to demonstrate a particular data analysis skill (e.g., box-and-whisker plots); rather, the purpose is the drawing of conclusions in a realistic setting using a range of techniques. **Make reasoned estimates:** These tasks require students to make reasonable estimates of things they do know, so that they can then build a chain of reasoning that gives them an estimate of something they *do not know*. **Plan and design:** Students recognize that this is a problem situation that arises in life and work. Well-posed planning tasks involving the coordinated analysis of time, space, and cost have already been recommended for assessing Claim 2. For Claim 4, the problem will be presented in a more open form, asking the students to identify the variables that need to be taken into account and the information they will need to find. **Evaluate and recommend:** These tasks involve understanding a model of a situation and/or some data about it and making a recommendation. **Interpret and critique:** These tasks involve interpreting some data and critiquing an argument based on it. Again, the purpose of these tasks is not to provide a setting for the student to demonstrate a particular data analysis skill, but to draw conclusions in a realistic setting using a range of techniques. Note: This is not a complete list; other types of tasks that fit the criteria above may well be included. *[Writers/developers should look for other kinds of evidence that support the Claim 4 targets.]*
Allowable Tools:	protractor, ruler, spreadsheet, mathematical software
Key Nontargeted Constructs:	*[Note: This cell identifies knowledge and skills the student needs in order to respond, but which are not scored for specified target(s).]*
Claim-Specific Attributes:	CR, ER, and TE tasks should be designed to take 10–20 minutes to solve, while PTs may take up to 120 minutes. A key feature of Claim 4 applied tasks is that the student must solve real-world problems that are not well-formulated in mathematical terms and/or are nonroutine. Additionally, students can be expected to make decisions about which information is relevant and how it should be represented. The computational demand of these tasks should focus on the skill

	level typically expected for Claim 1 tasks for grades lower than Grade [3, 4, 5] (though not to the exclusion of Grade [3, 4, 5] skills).
Accessibility Concerns:	Problems that require students to communicate reasoning may sometimes be text heavy. Translation tools and dictionaries should be available to ELL students. Text readers should be available to students as necessary.
Sample Items:	*[Note: Item codes will be hot-button links to sample items that illustrate Claim 4.]*

*SR = selected-response item; CR = constructed-response item; TE = technology-enhanced item; ER = extended-response item; PT = performance task

How to Complete the Item Form for Claim 4

Tasks that are written to Claim 4 assessment targets must follow the guidelines contained throughout these specifications. Additionally, item writers must complete an Item Form for every submitted task. Figure 11 provides the model used for all Claim 4 tasks, along with an explanation of the metadata that populates each cell of the form.

Figure 11.

Sample Item ID:	MAT.GR.IT.4.CDOMA.T.xxx
Grade:	Specify the 2-digit grade level (HS for high school).
Primary Claim:	**Claim 4: Modeling and Data Analysis** Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.
Secondary Claim(s):	List Claim 1 targets first, then Claim 2 or 3 targets (as needed), in order of prominence. May be left blank.
Primary Content Domain:	List the primary content domain of the task, as specified in the CCSSM.
Secondary Content Domain(s):	List additional content domains of the given task, as needed. May be left blank.
Assessment Target(s):	Multiple targets should be listed in order of prominence as related to the task. List the claim number first, then the target associated with that claim, accompanied by the text of the target (e.g., 4 D: Interpret results in the context of a situation.)
Standard(s):	Enter the number(s) of the CCSSM standard(s).
Mathematical Practice(s):	Specify the mathematical practices (1–8) associated with the item/task.
DOK:	Specify the Depth of Knowledge level (1–4) of the item/task.
Item Type:	Specify the item type (CR, TE, ER, PT).
Score Points:	Specify the total point value of the item.
Difficulty:	Specify the estimated difficulty of item (L=Low, M=Medium, H=Hard). See below for further explanation of coding.
Key:	Specify "See Sample Top-Score Response" for Claim 4 tasks.
Stimulus/Source:	Specify any stimulus material used and/or source required for factual information. All sources must be reliable and reproducible. If none, leave blank.
Claim-Specific Attributes (e.g., accessibility issues):	Specify any target-specific attributes (e.g., accessibility issues).
Notes:	Add any notes here that you believe will aid in understanding the purpose of this sample item. For TE items, include the TE template name here.

These are all intended to be "hot-button" links to the complete text of each component.

Sample Item ID: Specify the sample item ID "MAT.GR.IT.C.CDOMA.T.xxx"

MAT – Mathematics
GR – 03, 04, 05, 06, 07, 08, or HS
IT – Item type (SR, CR, TE, ER, or PT)
C – Claim number 1, 2, 3, or 4
CDOMA – Content Domain letters from CCSS (e.g., OA, MD, …must be five places, lead with zeros until all five places are filled)

Note: For PTs, the content domain field is filled with the task name abbreviation.

T – Assessment target alpha (A, B, C, D, etc.)
xxx – Item number. Leave alone for now; will be assigned after acceptance.

Difficulty – Base the level on the percent of students that would be expected to get the item/task correct or to earn the maximum number of points as follows:
Low – greater than 70% Medium – 40% to 70% Hard – less than 40%

Item/Task:

Use this space for the stem, stimulus, and/or options. The font size for items/tasks is Verdana, 14-point.

Sample Top-Score Response:

Provide an example of a complete and thorough top-score response. The language of this response does not need to be "kid-speak," but it should model what is expected from a student at the specified grade.

Scoring Rubric:

The scoring rubric for each task should reflect the values set out for this claim, giving substantial weight to the choice of appropriate methods of attacking the task, to reliable skills in carrying it through, and to explanations of what has been found.

Additionally, the language of the rubric should:

- focus on the essence of the primary claim;
- address the requirements of the specific target(s);
- distinguish between different levels of understanding and/or performance; and
- contain relevant information/details/numbers that support different levels of competency related to the item/task.

Appendix A: *Smarter Balanced* Mathematics Glossary

Grades 3–High School

Many of the terms and definitions in this glossary have been taken directly from the ***Common Core State Standards for Mathematics*** (CCSSM) and are indicated with an asterisk (*). The original CCSSM glossary has been supplemented with definitions to aid in the interpretation of the specification tables and item forms found throughout these *Specifications*. Italicized words or phrases within a definition are defined separately in this glossary.

The Claim 1 Mathematics Specification Tables contain a cell for "allowable disciplinary vocabulary" for each target, by grade. All vocabulary terms referenced in a previous grade are considered expected knowledge for subsequent grades.

Absolute value A number's distance from zero (0) on a number line. Distance is expressed as a positive value. Example: |7| = 7 and |-7| = 7.

Acute angle An *angle* that measures less than 90° and greater than 0°.

Addend Any number being added.

Additive identity The number zero (0). When zero (0) is added to another number, it does not change the number's value (e.g., 12 + 0 = 12).

Additive Two numbers whose sum is 0 are additive inverses of **inverses*** one
another. Example: ¾ and - ¾ are additive inverses
of one another because ¾ + (- ¾) = 0.

Algebraic A mathematical sentence containing *variables* in which **equation** two
expressions are connected by an equality **(inequality)** (inequality) symbol. See also:
equation and *inequality*.

Algebraic An expression containing numbers and *variables* (e.g., **expression**
3*x*), and operations that involve numbers and *variables*
(e.g., $8x + 5y$ or $4a^2 - 7b + 13$). Algebraic expressions
do not contain equality or *inequality* symbols.

Algebraic rule A mathematical *expression* that contains *variables* and describes a pattern or relationship.

Altitude The *perpendicular* distance from a *vertex* in a *polygon* to its opposite side.

Angle Two *rays* extending from a common end point called the *vertex*. Angles are measured in degrees.

Area	The measure, in square units, of the inside region of a closed two-dimensional figure (e.g., a rectangle with sides of 3 units by 9 units has an area of 27 square units).
Associative property	The way in which three or more numbers are grouped for addition or multiplication does not change their *sum* or *product*, respectively [e.g., (3 + 1) + 9 = 3 + (1 + 9) or (5 X 3) X 10 = 5 X (3 X 10)].
Axes (of a graph)	The horizontal and vertical *number lines* used in a *coordinate plane system*.
Axis	The singular form of *axes*.
Bar graph	A graph that uses either vertical or horizontal bars to *display data*.
Base (algebraic)	The number used as a factor in *exponential form*. Example: 5^3 is the *exponential form* of 5 X 5 X 5. The numeral five (5) is called the base, and the numeral three (3) is called the *exponent.*
Base (geometric)	The line or plane of a geometric figure, from which an *altitude* can be constructed, upon which a figure is thought to rest.
Bivariate data*	Pairs of linked numerical observations. Example: a list of heights and *weights* for each player on a football team.
Box plot*	A method of visually displaying a distribution of data values by using the *median*, quartiles, and extremes of the data set. A box shows the middle 50% of the data.[12]
Break	A zigzag on the *x*- or *y*-axis in a line or bar graph indicating that the data being displayed do not include all of the values that exist on the *number line* used. Also called a *squiggle*.
Capacity	The amount of space that can be filled in a container. Both capacity and *volume* are used to measure three-dimensional spaces; however, capacity usually refers to fluids, whereas *volume* usually refers to solids.
Central angle	An angle that has its *vertex* at the center of a circle, with *radii* as its sides.
Circle graph	A *data display* that divides a circle into regions representing a portion of the total set of data. The circle represents the whole set of data.
Circumference	The *perimeter* of a circle.

[12] Adapted from Wisconsin Department of Public Instruction, http://dpi.wi.gov/standards/mathglos.html, accessed March 2, 2010.

Closed figure A two-dimensional figure that divides the *plane* in which the figure lies into two parts—the part inside the figure and the part outside the figure (e.g., circles, squares, rectangles).

Commutative The order in which two numbers are added or multiplied **property** does not change their *sum* or *product*, respectively (e.g., 4 + 3 = 3 + 4 or 5 X 7 = 7 X 5).

Complementary angles Two *angles* with measures that sum to be exactly 90°.

Complex fraction* A fraction *A/B* where *A* and/or *B* are fractions (*B* nonzero).

Composite number A whole number that has more than two *factors*.

Congruent* Two plane or solid figures are congruent if one can be obtained from the other by *rigid motion* (a sequence of *rotations, reflections*, and *translations*).

Coordinate grid or plane A two-dimensional network of horizontal and vertical lines that are *parallel* and evenly-spaced; especially designed for locating points, displaying data, or drawing maps.

Coordinates Numbers that correspond to points on a *coordinate plane* in the form (*x, y*), or a number that corresponds to a point on a *number line*.

Counting principle If a first event has *n* outcomes and a second event has *m* outcomes, then the first event followed by the second event has *n* X *m* outcomes.

Customary units The units of measure developed and used in the United States. Customary units for:

- *length* are inches, feet, yards, and miles.
- *weight* are ounces, pounds, and tons.
- *volume* are cubic inches, cubic feet, and cubic yards.
- *capacity* are fluid ounces, cups, pints, quarts, and gallons.

Cylinder A three-dimensional figure with two *parallel* bases that are *congruent* circles.

Data displays/ Different ways of displaying data in *charts, tables*, or **graphs** *graphs*, including *pictographs, circle graphs*, single-, double-, or triple-*bar* and *line graphs, histograms, stem-and-leaf plots, box-and-whisker plots*, and *scatter plots*.

Decimal number Any number written with a decimal point in the number. A decimal number falls between two *whole numbers* (e.g., 4.7 falls between 5 and 6). Decimal numbers smaller than 1 are sometimes called decimal fractions (e.g., three-tenths is written 0.3).

Diameter A *line segment* from any point on the circle passing through the center to another point on the circle.

Difference A number that is the result of subtracting two numbers.

Dilation* A *transformation* that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common *scale factor.*

Direct measure Obtaining the measure of an object by using measuring devices, either standard devices of the *customary* or *metric systems*, or nonstandard devices such as a paper clip or pencil.

Distributive The *product* of a number and the *sum* or *difference* of **property** two
numbers is equal to the *sum* or *difference* of the
two *products*. Example: $x(a + b) = ax + bx$.

Divisible A number capable of being divided into equal parts without a remainder.

Divisor The number by which another number is divided.

Dot plot* See: *line plot.*

Empirical The likelihood of an event happening that is based on **probability** experience and observation rather than on theory.

Enlargement See: *dilation.*

Equation A mathematical sentence in which two *expressions* are connected by an equality symbol. See also *algebraic equation (inequality*).

Equilateral triangle A triangle with three *congruent* sides.

Equivalent *Expressions* that have the same value but are presented **expressions** in a different format using the properties of numbers.

Equivalent forms The same number expressed in different forms (e.g., ¼, **of a number** 0.25, 25%).

Estimation The use of rounding and/or other strategies to determine a reasonably accurate approximation, without calculating an exact answer (e.g., clustering, front-end estimating, grouping).

Evaluate an Substitute numbers for the *variables* and follow the **algebraic**
algebraic order of operations to find the numerical value
expression of the *expression*.

Expanded form* A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten (e.g., 643 = 600 + 400 + 3).

Expected value*	For a random *variable,* the weighted average of its possible values, with *weights* given by their respective probabilities.
Exponent	The number of times the *base* occurs as a *factor.* (exponential form) Example: 5^3 is the exponential form of 5 X 5 X 5. The numeral five (5) is called the base, and the numeral three (3) is called the exponent.
Expression	A collection of numbers, symbols, and/or operation signs that stands for a number.
Extraneous	Information that is not necessary to solving the **information** problem.
Extrapolate	To *estimate* or infer a value or quantity beyond the known range of data.
Face	One of the *plane* surfaces bounding a three-dimensional figure; a side.
Factor	A number or *expression* that divides evenly into another number [e.g., 1, 2, 4, 5, 10, and 20 are factors of 20 and (x + 1) is one of the factors of (x^2 – 1)].
First quartile*	For a data set with *median M*, the first quartile is the *median* of the data values less than *M*. Example: for the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6. See also: *median, third quartile,* and *interquartile range*.
Fraction*	A number expressible in the form *a*/*b* where *a* is a *whole number* and *b* is a positive *whole number*. (The word *fraction* in these specifications always refers to a non-negative number.) See also: *rational number.*
Function	A *relation* in which each value of *x* is paired with a unique value of *y*.
Function table	A table of *x*- and *y*-values (*ordered pairs*) that represents the *function*, *pattern*, *relationship*, or *sequence* between the two *variables*.
Grid	See: *coordinate grid.*
Height	A *line segment* extending from the *vertex* or apex of a figure to its *base* and forming a *right angle* with the *base* or the *plane* that contains the *base*.
Hypotenuse	The longest *side* of a right triangle; the *side* opposite the *right angle*.
Hypothesis	A proposition or supposition developed to provide a basis for further investigation or research.

Independently combined models* Two *probability* models are said to be combined independently if the *probability* of each *ordered pair* **probability** in the combined model equals the *product* of the original probabilities of the two individual outcomes in the *ordered pair*.

Indirect measure The measurement of an object through the known measure of another object.

Inequality A sentence that states one *expression* is greater than, greater than or equal to, less than, less than or equal to, or not equal to, another *expression* (e.g., $a \neq 2$ or $x < 4$ or $3y + 5 \geq 12$). See also *algebraic inequality*.

Integer* A number expressible in the form *a* or *-a* for some whole number *a*.

Intercept The value of a *variable* when all other *variables* in the *equation* equal zero (0). On a graph, the values where a *function* crosses the *axes*.

Interquartile A measure of variation in a set of numerical data, the **range*** interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9. See also: *first quartile, third quartile.*

Intersection The point at which two lines meet.

Inverse operation An action that undoes a previously applied action. Example: subtraction is the inverse operation of addition.

Irrational number A *real number* that cannot be expressed as a *ratio* of two numbers (e.g., $\sqrt{3}$).

Isosceles triangle A triangle with two *congruent* sides and two *congruent angles*.

Labels (for a graph) The titles given to a graph, the *axes* of a graph, or to the *scales* on the *axes* of a graph.

Length A one-dimensional measure that is the measurable property of *line segments*.

Likelihood The chance that something is likely to happen. See: *probability*.

Line A collection of an infinite number of *points* in a straight pathway with unlimited *length* and having no width.

Line plot* A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. Also known as a dot plot.[13]

Linear equation An *algebraic equation* in which the *variable* quantity or quantities are in the first power only and the graph is a straight *line.* Examples: $40 = 5(x + 1) + 2y$; $y = 6x + 11$].

Linear inequality An *algebraic inequality* in which the *variable* quantity or quantities are in the first power only and the graph is a region whose boundary is the straight *line* formed by the inequality.

Line graph A graph that displays data using connected *line segments*.

Line segment A portion of a *line* that consists of a defined beginning and endpoint and all the points in between.

Mass The amount of matter in an object.

Mean* A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.[14] Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Mean absolute A measure of variation in a set of numerical data, **deviation*** computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: for the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

Median* A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list — or the *mean* of the two central values, if the list contains an even number of values. Example: for the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

Metric units The units of measure developed in Europe and used in most of the world. Like the decimal system, the metric system uses the *base* 10. Metric units for:

- *length* are millimeters, centimeters, meters, and kilometers.
- *mass* are milligrams, grams, and kilograms.
- *volume* are cubic millimeters, cubic centimeters, and cubic meters.
- *capacity* are milliliters, centiliters, liters, and kiloliters.

Midline* In the graph of a trigonometric function, the horizontal line halfway

[13] Adapted from Wisconsin Department of Public Instruction, *op. cit.*
[14] To be more precise, this defines the *arithmetic mean*.

between its maximum and minimum values.

Midpoint of a line segment The *point* on a *line segment* that divides it into two equal parts.

Multiples The numbers that result from multiplying a given *whole number* by the set of *whole numbers* (e.g., the multiples of 12 are 0, 12, 24, 36, 48, 60).

Multiplicative The number one (1). Multiplying by 1 does not change a **identity** number's value (e.g., 8 X 1 = 8).

Multiplicative Two numbers whose product is 1 are multiplicative **inverses*** inverses of one another. Example: 3/4 and 4/3 are multiplicative inverses of one another because 3/4 X 4/3 = 4/3 X 3/4 = 1.

Natural numbers (counting numbers) The numbers in the set {1, 2, 3, 4, 5 . . .}.

Negative exponent Used to designate the *reciprocal* of a number to the *absolute value* of the *exponent.* Also used in scientific notation to designate a number smaller than one (1). Example: 4.21 X 10^{-2} equals 0.0421.

Nonstandard units Objects such as blocks, paper clips, crayons, or pencils **of measure** that can be used to obtain a measure.

Number line A diagram of the number line used to represent **diagram*** numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

Obtuse angle An *angle* with a measure of more than 90° but less than 180°.

Odds The *ratio* of one event occurring (favorable outcome) to it not occurring (unfavorable outcome) if all outcomes are equally likely.

Operation Any mathematical process, such as addition, subtraction, multiplication, division, raising to a power, or finding the square root.

Ordered pair The location of a single *point* on a *rectangular coordinate system*, where the digits represent the position relative to the *x-axis* and *y-axis* [e.g., (*x, y*) or (1, -2)].

Organized data Data arranged in a display that is meaningful and assists in the interpretation of the data. See: *data displays*.

Origin The *point* of *intersection* of the *x*- and *y-axes* in a *rectangular coordinate system*, where the *x-coordinate* and *y-coordinate* are both zero (0).

Parallel lines	Two *lines* in the same *plane* that are a constant distance apart. Parallel lines never meet and have equal *slopes*.
Pattern (relationship)	A predictable or prescribed *sequence* of numbers, objects, etc. Patterns and relationships may be described or presented using manipulatives, *tables*, graphics (pictures or drawings), or *algebraic rules (functions)*.
Percent	A special-case *ratio* which compares numbers to 100 (the second term). Example: 75% means the *ratio* of 75 to 100.
Percent rate of change*	A rate of change expressed as a *percent*. Example: if a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year.
Perimeter	The distance around a *polygon*.
Perpendicular	Two *lines*, two *line segments*, or two *planes* that cross to form a *right angle*.
Pi (π)	The symbol designating the *ratio* of the *circumference* of a circle to its *diameter*. It is an *irrational number* with common approximations of either 3.14 or 22/7.
Place value	The position of a single digit in a number.
Planar cross-section	The intersection of a *plane* and a three-dimensional figure.
Plane	An undefined, two-dimensional geometric surface that has no depth and no boundaries specified. A plane is determined by defining at least three distinct *points* or at least two distinct *lines* existing on the plane.
Plane figure	A two-dimensional figure that lies entirely within a single *plane*.
Point	A specific location in space that has no discernible *length* or width.
Polygon	A closed-*plane* figure, having at least three sides that are *line segments* and are connected at their end-points.
Prime number	Any *whole number* with only two *whole number factors*, 1 and itself (e.g., 2, 3, 5, 7, 11).
Probability*	A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).
Probability distribution*	The set of possible values of a random *variable* with a

probability assigned to each.

Probability A probability model is used to assign probabilities to **model*** outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. See also: *uniform probability model.*

Product The result of multiplying numbers together.

Proof A logical argument that demonstrates the truth of a given statement. In a formal proof, each step can be justified with a reason; such as a given, a definition, an axiom, or a previously proven property or theorem.

Proportion A mathematical sentence stating that two *ratios* are equal.

Proportional Having the same or a constant *ratio*. Two quantities that have the same *ratio* are considered directly proportional. Example: if $y = kx$, then y is said to be directly proportional to x and the constant of proportionality is k. Two quantities whose *products* are always the same are considered inversely proportional. Example: if $xy = k$, then y is said to be inversely proportional to x.

Pyramid A three-dimensional figure whose *base* is a *polygon* and whose *faces* are triangles with a common *vertex*.

Pythagorean The square of the *hypotenuse* (c) of a *right triangle* is **theorem** equal to the sum of the square of the legs (a and b), as shown in the *equation* $c^2 = a^2 + b^2$.

Quadrant Any of the four regions formed by the *axes* in a *rectangular coordinate system*.

Quotient The result of dividing two numbers.

Radical An expression that has a root (e.g., square root, cube root). Example: $\sqrt{36}$ is a radical. Any root can be specified by an index number, b, in the form $\sqrt[b]{a}$ (e.g., $\sqrt[3]{27}$). A radical without an index number is understood to be a square root.

Radical sign The symbol ($\sqrt{\ }$) used before a number to show that the number is a *radicand*. See also: *radical.*

Radicand The number that appears within a *radical sign* (e.g., in $\sqrt{36}$, 36 is the radicand).

Radius A *line segment* extending from the center of a circle or *sphere* to a *point* on the circle or *sphere*. Plural: radii.

Randomly (chosen)	An equal chance of being chosen.
Range	The lowest value (L) in a set of numbers through the highest value (H) in the set. When the width of the range is expressed as a single number, the range is calculated as the difference between the highest and lowest values (H – L). Other presentations show the range calculated as (H – L + 1). Depending on the context, the result of either calculation would be considered correct.
Rate	A *ratio* that compares two quantities of different units (e.g., miles per hour).
Ratio	The comparison of two quantities (e.g., the ratio of *a* and *b* is a:b or *a/b*, where $b \neq 0$).
Rational	A *quotient* of two polynomials with a non-zero **expression*** denominator.
Rational number*	A number expressible in the form *a/b* or – *a/b* for some *fraction a/b*. The rational numbers include the *integers*.
Ray	A portion of a *line* that begins at an endpoint and goes on indefinitely in one direction.
Real numbers	The set of all *rational* and *irrational numbers*.
Reciprocal	See: *multiplicative inverse*.
Rectangular coordinate system	See: *coordinate grid or plane.*
Rectilinear figure*	A *polygon*, all angles of which are right *angles*.
Reduction	See: *dilation.*
Reflection	A *transformation* that produces the mirror image of a geometric figure over a *line* of reflection.
Reflexive property	A number or *expression* is equal to itself (e.g., 3 = 3 or **of equality** *ab* = *ab*).
Regular polygon	A *polygon* that is both equilateral (all *sides congruent*) and equiangular (all *angles congruent*).
Relation	A set of *ordered pairs* (*x, y*).
Relative size	The size of one number in comparison to the size of another number or numbers.
Repeating	The decimal form of a *rational number*. See also: **decimal***

terminating decimal.

Right angle An *angle* whose measure is exactly 90º.

Right circular A *cylinder* in which the *bases* are *parallel* circles, **cylinder** *perpendicular* to the *side* of the *cylinder*.

Right prism or A three-dimensional figure (polyhedron) with **rectangular solid** *congruent,* polygonal *bases* and lateral *faces* that are all parallelograms.

Rigid motion* A *transformation* of points in space consisting of a sequence of one of more *translations, reflections*, and/or *rotations*. Rigid motions here are assumed to preserve distances and *angle* measures.

Rotation A *transformation* of a figure by turning it about a center *point* or *axis*. The amount of rotation is usually expressed in the number of degrees (e.g., a 90º rotation).

Rule A mathematical *expression* that describes a *pattern* or *relationship*, or a written description of the *pattern* or *relationship*.

Sample space* In a *probability model* for a random process, a list of the individual outcomes that are to be considered.

Scale The numeric values, set at fixed intervals, assigned to the *axes* of a *graph*.

Scale factor The constant that is multiplied by the *length* of each *side* of a figure that produces an image that is the same shape as the original figure, but a different size.

Scale model A model or drawing based on a *ratio* of the dimensions for the model and the actual object it represents.

Scalene triangle A triangle having no *congruent* sides.

Scatter plot* A graph in the *coordinate plane* representing a set of *bivariate data*. For example, the heights and *weights* of a group of people could be displayed on a scatter plot.[15]

Scientific notation A shorthand method of writing very large or very small numbers using *exponents* in which a number is expressed as the *product* of a power of 10 and a number that is greater than or equal to one (1) and less than 10 (e.g., $4.23 \times 10^6 = 4{,}230{,}000$).

Sequence An ordered list of numbers with either a constant *difference* (arithmetic) or a constant *ratio* (geometric).

[15] Adapted from Wisconsin Department of Public Instruction, *op. cit.*

Side	The edge of a *polygon* (e.g., a triangle has three *sides*) or one of the *rays* that make up an *angle*.
Similar figures	Figures that are the same shape, have corresponding, *congruent angles*, and have corresponding *sides* that are *proportional* in *length*.
Similarity	A term describing figures that are the same shape but are not necessarily the same size or in the same position.
Slope	The *ratio* of change in the vertical *axis* (*y*-axis) to each unit change in the horizontal *axis* (*x*-axis) in the form rise/run or $\Delta x/\Delta y$. Also, the constant, *m,* in the linear *equation* for the *slope-intercept* form, $y = mx + b$.
Solid figures	Three-dimensional figures that completely enclose a portion of space (e.g., a rectangular solid, cube, *sphere*, *right circular cylinder*, *right circular cone*, and *square pyramid*).
Sphere	A three-dimensional figure in which all *points* on the figure are equidistant from a center *point*.
Square root	A positive *real number* that can be multiplied by itself to produce a given number (e.g., the square root of 49 is 7 or $\sqrt{49} = 7$).
Squiggle	See: *break*.
Standard units	Accepted measuring devices and units of the *customary* **of measure** or *metric system*.
Stem-and-leaf plot	A graph that organizes data by place value to compare data frequencies.
Straight angle	An *angle* that measures exactly 180°.
Sum	The result of adding numbers together.
Supplementary angles	Two *angles*, the *sum* of which is exactly 180°.
Surface area of a	The *sum* of the *areas* of the *faces* and any curved **geometric solid** surfaces of the figure that create the geometric solid.
Symmetry	A term describing the result of a *line* drawn through the center of a figure such that the two halves of the figure are *reflections* of each other across the *line*.
System of equations	A group of two or more *equations* that are related to the same situation and share *variables*. The solution to a system of equations is an ordered number set that makes all of the *equations* true.

Tape diagram* A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, *fraction* strip, or *length* model.

Terminating decimal* A decimal is called terminating if its repeating digit is 0.

Theoretical/expected probability The *likelihood* of an event happening based on theory rather than on experience and observation.

Third quartile* For a data set with *median M*, the third quartile is the *median* of the data values greater than *M*. Example: for the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. See also: *median, first quartile, interquartile range*.

Transformation An *operation* on a geometric figure by which another image is created. Common transformations include *reflections* (*flips*), *translations* (*slides*), and *rotations* (*turns*) and dilations.

Transitive property When the first element has a particular relationship to a second element that in turn has the same relationship to a third element, the first has this same relationship to the third element (e.g., if $a = b$ and $b = c$, then $a = c$).

Transitivity principle for indirect measurement* If the *length* of object *A* is greater than the *length* of object *B*, and the *length* of object *B* is greater than the *length* of object *C*, then the *length* of object *A* is greater than the *length* of object *C*. This principle applies to measurement of other quantities as well.

Translation A *transformation* in which every *point* in a figure is moved in the same direction and by the same distance.

Transversal A *line* that *intersects* two or more *lines* at different *points*.

Tree diagram A diagram in which all the possible outcomes of a given event are displayed.

Trend line A *line* on a *graph* that shows a trend between data *points*.

Uniform probability model* A *probability model* which assigns equal *probability* to all outcomes. See also: *probability model.*

Unorganized data Data that are presented in a *random* manner.

Variable Any symbol, usually a letter, that could represent a number.

Vector* A quantity with magnitude and direction in the *plane* or in space defined by an *ordered pair* or triple of *real numbers*.

Vertex	The common endpoint from which two *rays* begin (i.e., the vertex of an *angle*) or the *point* where two *lines intersect*; the *point* on a triangle or *pyramid* opposite to and farthest from the base.
Vertical angles	The opposite or non-adjacent *angles* formed when two *lines intersect*.
Visual fraction model*	A *tape diagram*, number line diagram, or *area* model.
Volume	The amount of space occupied in three dimensions and expressed in cubic units. Both *capacity* and *volume* are used to measure empty spaces; however, *capacity* usually refers to fluids, whereas *volume* usually refers to solids.
Weight	Measures that represent the force of gravity on an object.
Whole numbers	The numbers in the set {0, 1, 2, 3, 4 . . .}.
***x*-axis**	The horizontal *number line* on a *rectangular coordinate system*.
***x*-intercept**	The value of *x* at the *point* where a *line* or *graph intersects* the *x-axis.* The value of *y* is zero (0) at this *point*.
***y*-axis**	The vertical *number line* on a *rectangular coordinate system*.
***y*-intercept**	The value of *y* at the *point* where a *line* or *graph intersects* the *y-axis.* The value of *x* is zero (0) at this *point*.

Appendix B: Cognitive Rigor Matrix/Depth of Knowledge (DOK)

The Common Core State Standards require high-level cognitive demand, such as asking students to demonstrate deeper conceptual understanding through the application of content knowledge and skills to new situations and sustained tasks. For each Assessment Target in this document, the depth(s) of knowledge (DOK) that the student needs to bring to the item/task has been identified, using the Cognitive Rigor Matrix shown below. This matrix draws from two widely accepted measures to describe cognitive rigor: Bloom's (revised) Taxonomy of Educational Objectives and Webb's Depth-of-Knowledge Levels. The Cognitive Rigor Matrix has been developed to integrate these two models as a strategy for analyzing instruction, for influencing teacher lesson planning, and for designing assessment items and tasks. (To download the full article describing the development and uses of the Cognitive Rigor Matrix and other support CRM materials, go to: http://www.nciea.org/publications/cognitiverigorpaper_KH11.pdf)

A "Snapshot" of the Cognitive Rigor Matrix (Hess, Carlock, Jones, & Walkup, 2009)

Depth of Thinking (Webb)+ Type of Thinking (Revised Bloom)	DOK Level 1 Recall & Reproduction	DOK Level 2 Basic Skills & Concepts	DOK Level 3 Strategic Thinking & Reasoning	DOK Level 4 Extended Thinking
Remember	• Recall conversions, terms, facts			
Understand	• Evaluate an expression • Locate points on a grid or number on number line • Solve a one-step problem • Represent math relationships in words, pictures, or symbols	• Specify, explain relationships • Make basic inferences or logical predictions from data/observations • Use models /diagrams to explain concepts • Make and explain estimates	• Use concepts to solve non-routine problems • Use supporting evidence to justify conjectures, generalize, or connect ideas • Explain reasoning when more than one response is possible • Explain phenomena in terms of concepts	• Relate mathematical concepts to other content areas, other domains • Develop generalizations of the results obtained and the strategies used and apply them to new problem situations
Apply	• Follow simple procedures • Calculate, measure, apply a rule (e.g., rounding) • Apply algorithm or formula • Solve linear equations • Make conversions	• Select a procedure and perform it • Solve routine problem applying multiple concepts or decision points • Retrieve information to solve a problem • Translate between representations	• Design investigation for a specific purpose or research question • Use reasoning, planning, and supporting evidence • Translate between problem & symbolic notation when not a direct translation	• Initiate, design, and conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results
Analyze	• Retrieve	• Categorize data,	• Compare	• Analyze multiple

	information from a table or graph to answer a question • Identify a pattern/trend	figures • Organize, order data • Select appropriate graph and organize & display data • Interpret data from a simple graph • Extend a pattern	information within or across data sets or texts • Analyze and draw conclusions from data, citing evidence • Generalize a pattern • Interpret data from complex graph	sources of evidence or data sets
Evaluate			• Cite evidence and develop a logical argument • Compare/contrast solution methods • Verify reasonableness	• Apply understanding in a novel way, provide argument or justification for the new application
Create	• Brainstorm ideas, concepts, problems, or perspectives related to a topic or concept	• Generate conjectures or hypotheses based on observations or prior knowledge and experience	• Develop an alternative solution • Synthesize information within one data set	• Synthesize information across multiple sources or data sets • Design a model to inform and solve a practical or abstract situation